Online Advanced Study Program on Helicities in Astrophysics and Beyond
(Helicity 2020)

Compressible 'helical' turbulence: 'Fastened'-structure geometry and statistics

扎紧湍流：

可压缩"有螺"湍流："扎紧"流动的几何与统计

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Outline

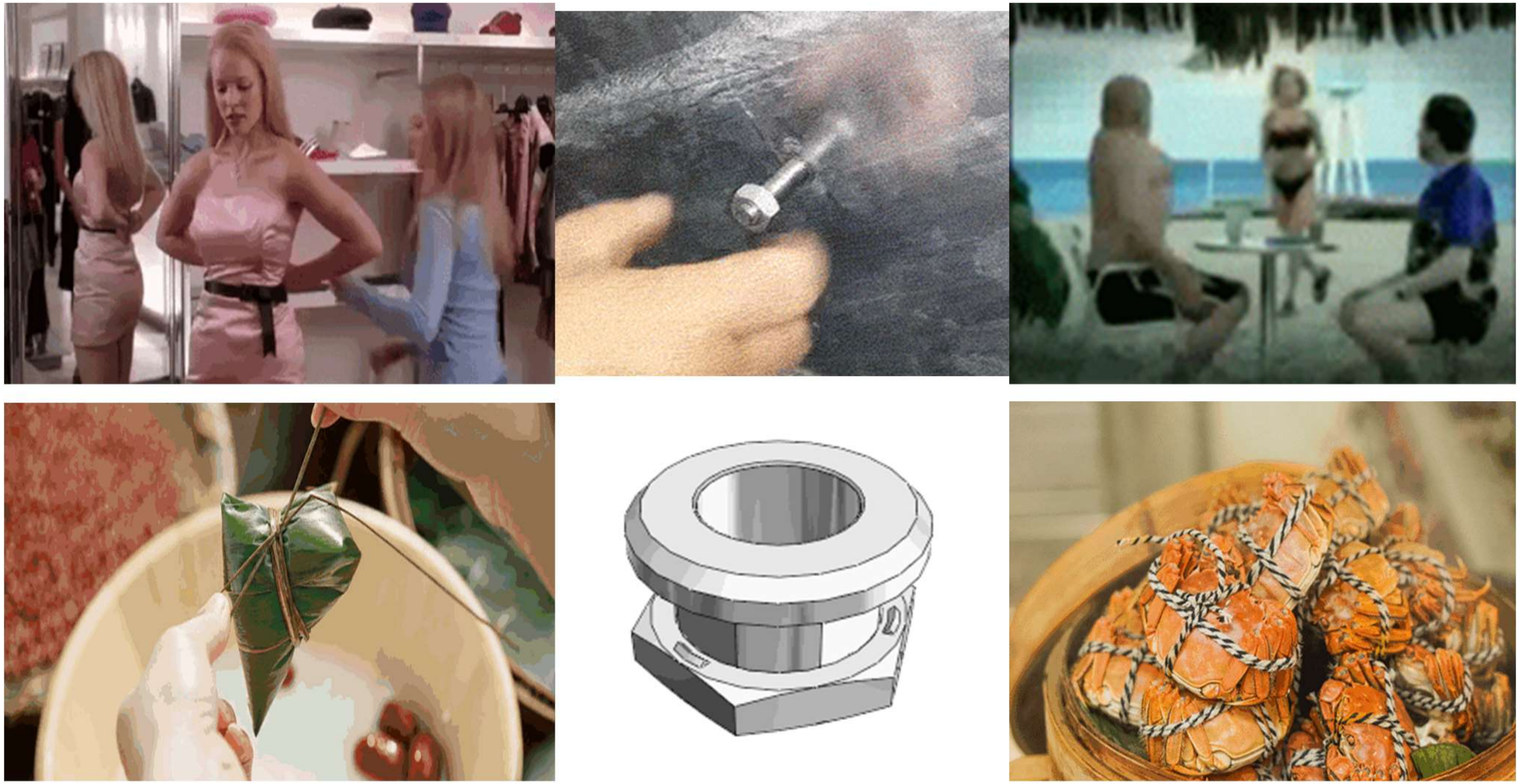
- 1. A fastened "chiral/helical base flow/field (CBF)" structure: the geometry of Taylor-Proudman effect: not that usually understood (扎紧"手性基本流/场"：非通常所理解之处)

- 2. Fastened turbulence: statistical constraints of helicities of various gas models (螺度的统计约束扎紧湍流)

- 3. Preliminary numerical evidence of **compressibility reduction ('fastened') with helicity** (初步数值验证)

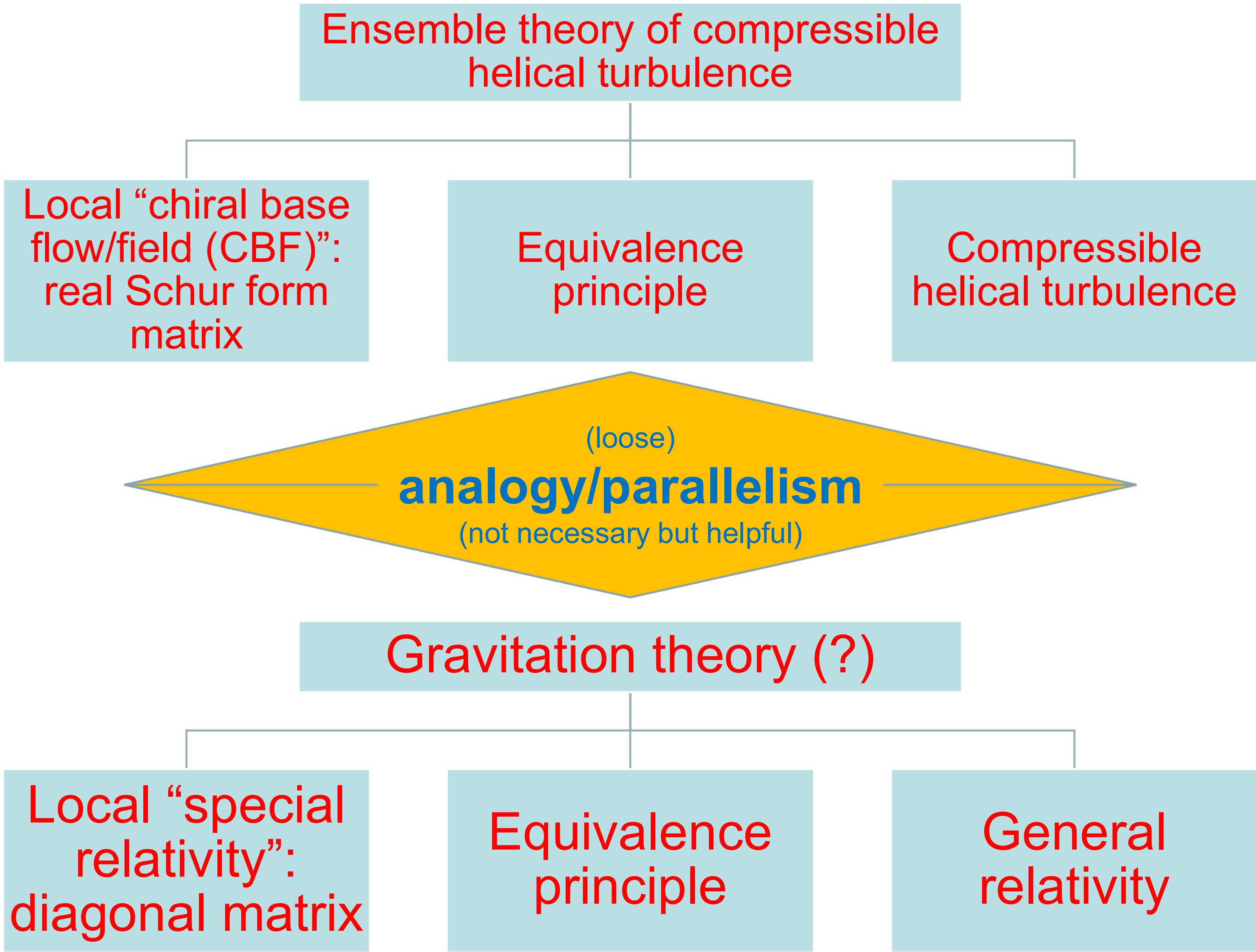
- 4. Bridging (fastened) CBF and (fastened) helical turbulence: ensemble theory: the equivalence principle
  （从结构到统计：系综理论：等效原理）

The notion of “fasten” “扎紧”之概念



夫“扎紧” 者，出于生产之用，入乎食色之需，乃日常观念也

logic/framework（大框架）



Part 1:

Fastened

“chiral/helical base flow/field (CBF)”:

boost to a rotating frame:

the geometry of

Taylor-Proudman effect:

not that usually understood

(扎紧“手性基本流/场”：TPE非通常所理解之处)

Local chiral/helical base flow/field (CBF)

[c.f., J.-Z. Zhu, Phys. Fluids **30**, 031703 (2018)]

$$\nabla_{\boldsymbol{r}} \boldsymbol{u} = \begin{pmatrix} u_{x,x} & u_{y,x} & u_{z,x} \\ u_{x,y} & u_{y,y} & u_{z,y} \\ \cancel{u_{x,z}} & \cancel{u_{y,z}} & u_{z,z} \end{pmatrix}$$

Real Schur form (RSF)

[5]C. J. Keylock and S. Tian *et al.* have applied RSF in their respective studies of flow structures (private communications, 2017 and 2018).

[6]The local RSF could be helpful for clarifying the *global regularity* issue. And, this is reminiscent of the locally inertial coordinates, supporting the *principle of equivalence* in general relativity. We can also work with other vector gradients, whose antisymmetric parts correspond to the magnetic field, the electric current or else, and may formulate a kind of principle of equivalence for the specific dynamics.

局部手性基本流/场

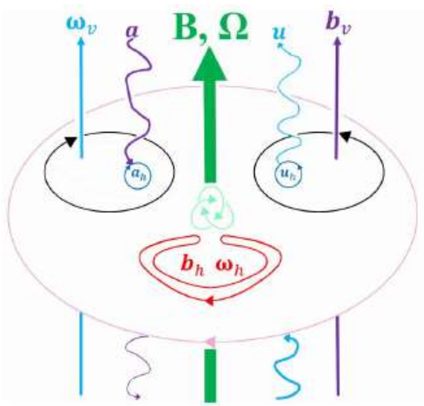
$$u_{x,z} = 0 = u_{y,z}, \tag{6}$$

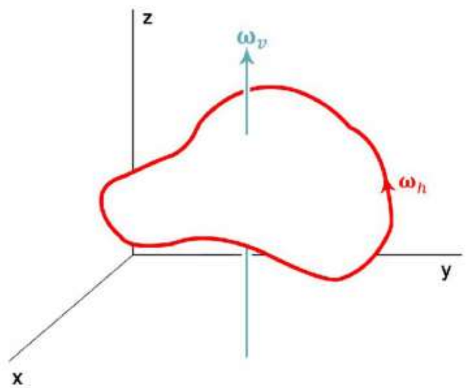
with which locally two-component-two-dimensional coupled-with-one-component-three-dimensional (2C2Dcw1C3D) flow results, and with which we here also call G an RSF. Then, from Eq. (4), we now have

$$\omega^{(1)} = (0, 0, u_{y,x} - u_{x,y}), \tag{7}$$

and, as in Eqs. (2) and (3),

$$\omega^{(2)} = (u_{z,y}, -u_{z,x}, 0) = \omega_3. \tag{8}$$





Boost to the rotating frame

(提升到旋转标架)

tegration by parts. Now, for given $\mathcal{H}$, we introduce such 'mean rotation' rate $\boldsymbol{\Omega}$ along the vertical direction that, for the relative velocity

$$\boldsymbol{u}' = \boldsymbol{u} - \boldsymbol{\Omega} \times \boldsymbol{r} = (u_x + y\Omega, u_y - x\Omega, u_z)$$

[thus $x' = x + y\Omega t$, $y' = y - x\Omega t$ and $z' = z$], the helicity of $\boldsymbol{\omega}' = \nabla \times \boldsymbol{u}'$ vanishes, *i.e.*,

$$\mathcal{H}' = \frac{1}{2\mathcal{V}} \int \boldsymbol{\omega}' \cdot \boldsymbol{u}' d^3\boldsymbol{r} = 0,$$

with $\boldsymbol{\Omega}$ determined, from

$$\boldsymbol{\omega} \cdot \boldsymbol{u} - \boldsymbol{\omega}' \cdot \boldsymbol{u}' = \boldsymbol{\Omega} \cdot [2\boldsymbol{u}' + (\boldsymbol{r} \times \boldsymbol{\omega}')] = \boldsymbol{\Omega} \cdot [2\boldsymbol{u} + (\boldsymbol{r} \times \boldsymbol{\omega})] \quad (16)$$

for general $\boldsymbol{u}$ and Eqs. (13,14) for CBF, by

$$\mathcal{H} = \boldsymbol{\Omega} \cdot \frac{\int [2\boldsymbol{u}_v + \boldsymbol{r} \times (\nabla \times \boldsymbol{u}_v)] d^3\boldsymbol{r}}{2\mathcal{V}}. \quad (17)$$

Our strategy is to reduce the relevant $\mathcal{H}$ effect on $\boldsymbol{u} = \boldsymbol{u}' + \boldsymbol{\Omega} \times \boldsymbol{r}$ to the $\boldsymbol{\Omega}$ effect on the relative motion with no helicity via "boost" to the rotating frame, for the CBF

the Taylor-Proudman effect of "fastening"

Taylor-Proudman 扎紧效应

(not two-dimensionalization)

In the CBF, for any material circuit $c(t)$ with horizontal projection area $\mathcal{A}(t) = \oint_{c(t)} xdy - ydx$, the circulation

$$\oint_{c(t)} \boldsymbol{u} \cdot d\boldsymbol{r} = \oint_{c(t)} \boldsymbol{u}' \cdot d\boldsymbol{r} + 2\Omega\mathcal{A} \quad (19)$$

is invariant, if the pressure term does not contribute (as in the barotropic case). If $\oint_c \boldsymbol{u}' \cdot d\boldsymbol{r}$ varies comparatively slowly, $\mathcal{A}$ changes little, which is the geometrical argument to prove the TPE by Taylor[42] for the two-dimensionalization of incompressible rotating flows.

knots "fasten" the gas. Indeed, $\mathcal{A} \to 0 \ \forall \ c(t)$ results in

$$\partial_z \boldsymbol{u}_h \to \boldsymbol{0} \text{ and } \nabla_h \cdot \boldsymbol{u}_h := \partial_x u_x + \partial_y u_y \to 0, \quad (20)$$

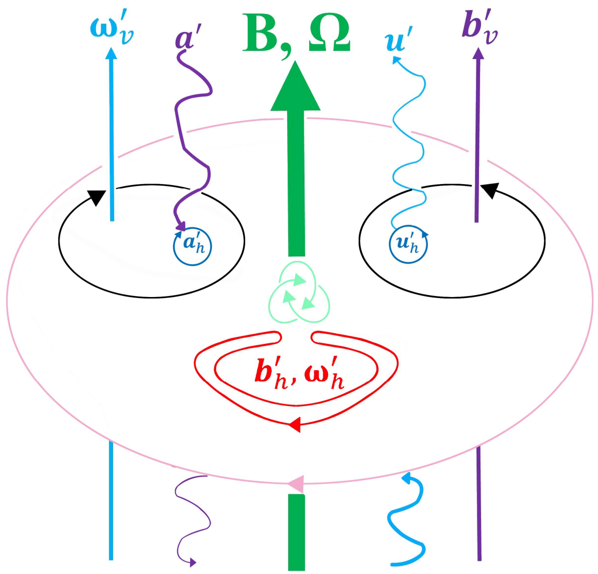
$$\text{but with no explicit constraint on } \partial_z u_z. \quad (21)$$

The latter half of Eq. (20) ~~which~~ implies that the horizontal compressibility $\nabla_h \cdot \boldsymbol{u}_h$ may be reduced by (finite) rotation, even for a time-dependent flow (Ref. [44], with analogues ~~[44]~~ for plasma flows and extensions for flows

The mechanics/geometry of Taylor-Proudman fastening effect

扎紧效应之流体力学/几何

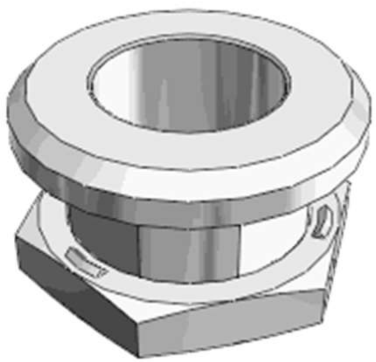
We now take the (projected) circuits in the compressible CBF to be any of the velocity streamlines *screwing* on the cylindrical surfaces and probably closing at infinity or at the periodic boundary with finite circulation in the *stream-screw scenario*,[43] or, to be any of the horizontal vorticity loops binding the vertical ones in the *vortex-knot scenario*,[25] both caricatured clearly in Fig. 1: $\mathcal{A}$ changing little geometrically demonstrates that the screws and knots "fasten" the gas. Indeed, $\mathcal{A} \to 0 \; \forall \; c(t)$ results in



“fasten” notion “扎紧”之概念








“扎紧” 者，出于生产之用，入乎食色之需，乃日常观念也

Part 2:

Indications of fastened turbulence:

statistical constraints of helicities

(螺度的统计约束扎紧湍流)

Aerodynamics (HD)

$$p = c^2\rho \text{ and } \rho = \rho_0 e^{\zeta} \text{ with an equilibrium } \rho_0.$$

The equations of motion then write

$$\partial_t \zeta + \zeta_{,\alpha} u_\alpha + u_{\alpha,\alpha} = 0, \tag{2}$$

$$\partial_t u_\lambda + u_\sigma u_{\lambda,\sigma} + c^2 \zeta_{,\lambda} - \eta \theta_{\lambda\sigma,\sigma} = 0, \tag{3}$$

where $\theta_{\alpha\beta} = u_{\alpha,\beta} + u_{\beta,\alpha} - \frac{2}{3}\delta^\alpha_\beta u_{\sigma,\sigma}$, $(\bullet)_{,\gamma} = \partial(\bullet)/\partial x^\gamma$. approximation for the ideal invariant of total, kinetic plus

internal/potential $\left(\int_{\rho_0}^{\rho} \frac{p - p_0}{\rho^2} d\rho \approx \frac{c^2\zeta^2}{2} \text{ for small } \zeta\right)$ (4)

mean energy per unit mass:

$$\mathcal{E} = \frac{\langle u^2 + c^2\zeta^2 \rangle}{2} = \frac{\sum_{\boldsymbol{k}} [\hat{u}_\lambda(\boldsymbol{k}) \hat{u}^*_\lambda(\boldsymbol{k}) + c^2 |\hat{\zeta}(\boldsymbol{k})|^2]}{2} \tag{5}$$

$$\dot{\hat{\zeta}}(\boldsymbol{k}) = \sum_{\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{k}} [\hat{\zeta}(\boldsymbol{p})\boldsymbol{p}\cdot\hat{\boldsymbol{u}}_{\perp}(\boldsymbol{q}) + \hat{\zeta}(\boldsymbol{p})\boldsymbol{p}\cdot\hat{\boldsymbol{u}}_{|}(\boldsymbol{q})] + k\hat{u}_{|}(\boldsymbol{k}), \tag{2.4}$$

$$\dot{\hat{\boldsymbol{u}}}_{|}(\boldsymbol{k}) = \hat{\boldsymbol{N}}_{|}(\boldsymbol{k}) + c^2\boldsymbol{k}\hat{\zeta}(\boldsymbol{k}), \tag{2.5}$$

$$\dot{\hat{\boldsymbol{u}}}_{\perp}(\boldsymbol{k}) = \hat{\boldsymbol{N}}_{\perp}(\boldsymbol{k}), \tag{2.6}$$

with $\boldsymbol{N} = \boldsymbol{u}\cdot\nabla\boldsymbol{u} = (\boldsymbol{u}_{|} + \boldsymbol{u}_{\perp})\cdot\nabla(\boldsymbol{u}_{|} + \boldsymbol{u}_{\perp})$, showing the complicated interactions (with, say, $\hat{\boldsymbol{N}}_{|}(\boldsymbol{k}) = (\widehat{\boldsymbol{u}_{\perp}\cdot\nabla\boldsymbol{u}_{\perp}})_{|} + (\widehat{\boldsymbol{u}_{|}\cdot\nabla\boldsymbol{u}_{|}})_{|} + (\widehat{\boldsymbol{u}_{\perp}\cdot\nabla\boldsymbol{u}_{|}})_{|} + (\widehat{\boldsymbol{u}_{|}\cdot\nabla\boldsymbol{u}_{\perp}})_{|}$), of the acoustic

Onsager, L. (1945: one line in a letter to Lin, C.-C.)

Lee, T.-D. (1952)

Kraichnan, R. H. (1955)

……

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 27, NUMBER 3 MAY, 1955

On the Statistical Mechanics of an Adiabatically Compressible Fluid*†

ROBERT H. KRAICHNAN

$$E = \frac{1}{2}\sum_k |\hat{u}_+|^2 + |\hat{u}_-|^2 + |\hat{u}_|\,|^2 + c^2|\hat{\zeta}|^2,$$

$$H = \frac{1}{2}\sum_k k|\hat{u}_+|^2 - k|\hat{u}_-|^2.$$

$$C = \alpha E + \beta H$$

$$\sim \exp\{-C\}$$

J. Fluid Mech. (2016), vol. 787, pp. 440–448. © Cambridge University Press 2015 440
doi:10.1017/jfm.2015.692

Isotropic polarization of compressible flows

Jian-Zhou Zhu[1,2,†]

P. 443:

$$\hat{\boldsymbol{u}}(\boldsymbol{k}) = \hat{u}_+(\boldsymbol{k})\hat{\boldsymbol{h}}_+(\boldsymbol{k}) + \hat{u}_-(\boldsymbol{k})\hat{\boldsymbol{h}}_-(\boldsymbol{k}) + \hat{u}_|(\boldsymbol{k})\boldsymbol{k}/k$$

$$U^+(\boldsymbol{k}) \triangleq \langle|\hat{u}_+|^2\rangle = \frac{1}{\alpha + \beta k}, \tag{2.8}$$

$$U^-(\boldsymbol{k}) \triangleq \langle|\hat{u}_-|^2\rangle = \frac{1}{\alpha - \beta k}, \tag{2.9}$$

$$Z(\boldsymbol{k}) \triangleq \langle c^2|\hat{\zeta}|^2\rangle = \frac{1}{\alpha}, \tag{2.10}$$

$$U^|(\boldsymbol{k}) \triangleq \langle|\hat{u}_||^2\rangle = \frac{1}{\alpha}. \tag{2.11}$$

We see that the spectra are directionally isotropic; but, when β or H is not zero, energies are polarized, except for the equipartition between the dilatational and density/pressure modes; and, the vortical-mode energy is larger than the 'acoustic' one,

$$U^\perp \triangleq U^+ + U^- > U^\sim \triangleq Z + U^|, \tag{2.12}$$

which indicates that helical states tend to have higher vorticity-mode energy and reduce 'noise'; see further discussions in the next section. (It is interesting, though

Such statistical dynamical (thermalization) mechanism and consequent property may be expected to persist for turbulent flows in which the energy damping and injection mechanisms are independent of helicity, but the discussions for cases with forcing and dissipation, where other effects enter (to assist or resist the mechanism) and other phenomena mingle (to encrease or decrease the property), should of course be careful, as performed

Note that the notion of 'flow compressibility' in this note is not that defined by $\partial\rho/\partial p$ through the state equation, the latter of a gas is not indicated to be directly changed when we say "compressibility reduction": We mean, even if the state equation is fixed, the strength of the fluctuations of velocity divergence and of density is (relatively) reduced.

Jian-Zhou Zhu, 2016, J. Fluid Mech., P. 445:

to a tendency of energy equipartition between the two modes', suggested by K55 as the authors explicitly referred to, may not be as favourable for strongly helical compressible turbulence. Our (2.12), $U^{\perp} > U^{\sim}$, in the AE state might be extrapolated to more general cases, which could be preliminarily seen by perturbing the last term of (2.2) and simply check the corresponding results. Unfortunately, it seems to us that no documentation of compressible helical turbulence data has been made available, and relevant *in-silico* or laboratory experiments are called for.

magnetoaerodynamics (MAD/MHD)

$$\partial_t \boldsymbol{b} - \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) = 0,$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + c^2 \nabla \zeta + \frac{\nabla \times \boldsymbol{b} \times \boldsymbol{b}}{\rho_0 \exp\{\zeta\}} = 0.$$

$$U_K^{\pm}(\boldsymbol{k}) := \langle |\hat{u}_{\pm}|^2 \rangle = \frac{4(\alpha\, k \pm \beta_M)}{(4\,\alpha^2 - {\beta_C}^2)k \pm 4\,\alpha\,\beta_M},$$

$$U_M^{\pm}(\boldsymbol{k}) := \langle |\hat{b}_{\pm}|^2 \rangle = \frac{4(\alpha\, k)}{(4\,\alpha^2 - {\beta_C}^2)k \pm 4\,\alpha\,\beta_M},$$

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$$E=\frac{1}{2}\sum_{\boldsymbol{k}}|\hat{u}_+|^2+|\hat{u}_-|^2+|\hat{u}_|\,|^2+c^2|\hat{\zeta}|^2,\qquad H=\frac{1}{2}\sum_{\boldsymbol{k}}k|\hat{u}_+|^2-k|\hat{u}_-|^2.\quad C=\alpha E+\beta H$$

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We see that the spectra are directionally isotropic; but, when β or H is not zero, energies are polarized, except for the equipartition between the dilatational and density/pressure modes; and, the vortical-mode energy is larger than the 'acoustic' one,

$$U^\perp\triangleq U^++U^->U^\sim\triangleq Z+U^|,\tag{2.12}$$

which indicates that helical states tend to have higher vorticity-mode energy and reduce 'noise'; see further discussions in the next section. (It is interesting, though

Extended magnetoaerodynamics: XMAD/XMHD

$$\partial_t \boldsymbol{u} = -\nabla\left[\Pi + u^2/2 + (d_e \nabla \times \boldsymbol{b})^2/(2\rho^2)\right] + \\ + \boldsymbol{u} \times (\nabla \times \boldsymbol{u}) + (\nabla \times \boldsymbol{b}) \times \ddot{\boldsymbol{b}}/\rho, \quad (22)$$

$$\partial_t \ddot{\boldsymbol{b}} = \nabla \times \left(\boldsymbol{u} \times \ddot{\boldsymbol{b}}\right) - d_i \nabla \times \left[(\nabla \times \boldsymbol{b}) \times \ddot{\boldsymbol{b}}/\rho\right] + \\ + d_e^2 \nabla \times \left[(\nabla \times \boldsymbol{b}) \times (\nabla \times \boldsymbol{u})/\rho\right], (23)$$

with d_i and d_e being the ion and electron skin depths, Π the enthalpy, and, for notational convenience,

$$\ddot{\boldsymbol{b}} := \boldsymbol{b} + d_e^2 \nabla \times (\nabla \times \boldsymbol{b}),\ \nabla \times \ddot{\boldsymbol{a}} = \ddot{\boldsymbol{b}}. \quad (24)$$

indeally invariant XMHD helicities, $\mathcal{H}_M := \langle \ddot{\boldsymbol{a}} \cdot \ddot{\boldsymbol{b}} + d_e^2 \boldsymbol{\omega} \cdot \boldsymbol{u} \rangle /2$ and $\mathcal{H}_C := \langle 2\boldsymbol{u} \cdot \ddot{\boldsymbol{b}} + d_i \boldsymbol{\omega} \cdot \boldsymbol{u} \rangle /2$, thus

$$\mathcal{H}_M = \frac{1}{2} \sum_{\boldsymbol{k},s} (skd_e^2 |\hat{u}^s|^2 + s|\hat{\ddot{b}}^s|^2/k), \tag{26}$$

$$\mathcal{H}_C = \frac{1}{2} \sum_{\boldsymbol{k},s} (\hat{u}^{s*} \hat{\ddot{b}}^s + c.c. + skd_i |\hat{u}^s|^2). \tag{27}$$

From the canonical distribution $\sim \exp\{-(\alpha \mathcal{E} + \beta_M \mathcal{H}_M + \beta_C \mathcal{H}_C)\}$, we have, besides the same Eq. (11) for density and parallel/compressive modes,

$$U_K^{\pm}(\boldsymbol{k}) = \frac{1}{\alpha \pm k(\beta_M d_e^2 + \beta_C d_i) - \frac{\beta_C^2}{\frac{\alpha}{1+k^2 d_e^2} \pm \frac{\beta_M}{k}}}, \tag{28}$$

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Two-fluid plasma model: Statistics

$\sim \exp\{-(\alpha\mathcal{E}+\beta_e\mathcal{H}_e+\beta_i\mathcal{H}_i)\}$ with the constraints of the (approximate) invariant energy

$$\mathcal{E} = \langle \boldsymbol{E}^2 + \boldsymbol{b}^2 + \sum_{\chi}(\rho_{0\chi}\boldsymbol{u}_{\chi}^2 + c_{\chi}^2\zeta_{\chi}^2)\rangle/2 \qquad (38)$$

and the self-helicities, defined by the canonical momenta $\boldsymbol{P}_\chi := m_{\not{s}\chi}\boldsymbol{u}_{\not{s}\chi} + q_\chi \boldsymbol{a}$,

$$\mathcal{H}_\chi := \langle \nabla \times \boldsymbol{P}_\chi \cdot \boldsymbol{P}_\chi\rangle/2\rangle \text{ for } \chi = i \text{ and } e, \qquad (39)$$

which can be represented, similar to Eqs. (8, 9), in the Fourier space with the Helmholtz and helical decompositions, resulting in, among others, the equipartitioned

$$Z_\chi(\boldsymbol{k}) := \langle c_\chi^2 |\hat{\zeta}_\chi|^2\rangle = \frac{1}{\alpha} = \langle \rho_{0\chi}|\hat{u}_{\chi|}|^2\rangle =: U^{|}_{\chi K}(\boldsymbol{k}), \quad (40)$$

$$\langle |\hat{E}|^2\rangle = \frac{1}{\alpha} \text{ or the finner } \langle |\hat{E}_{\pm}|^2\rangle = \langle |\hat{E}_{|}|^2\rangle = \frac{1}{\alpha}. \quad (41)$$

Two-fluid plasma model: geometry in the fastened electric field

The explicitly $\boldsymbol{E}$-related Maxwell equation with clear geometrical picture is the Gauss law, besides $\nabla \cdot \boldsymbol{b} = 0$,

$$\nabla \cdot \boldsymbol{E} = \sum_{\chi} \frac{q_\chi}{m_\chi} \rho_\chi \text{ or } \hat{E}_| = -\hat{i} \sum_{\chi} \frac{q_\chi}{m_\chi k_|} \hat{\rho}_\chi, \qquad (42)$$

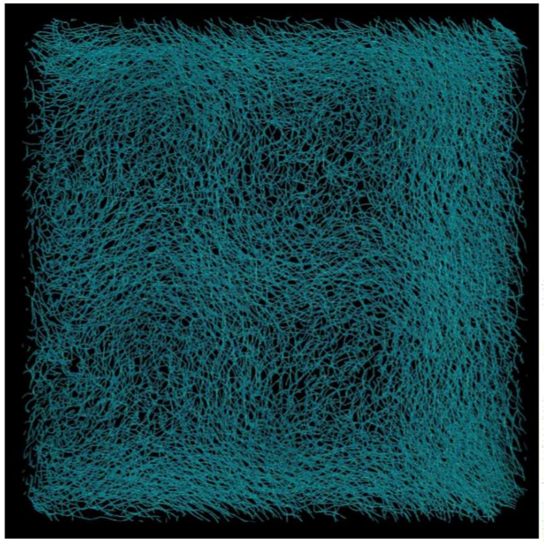
stating (with unit electric dielectric coefficient in the appropriate units and scales) that the electric displacement flux out of a surface enclosing a volume is the sum of the linear combination of masses (interpreted from the particle number densities for computing the total charge densities). If the density fluctuations of the two fluids are (approximately) independent, as is the case for the absolute-equilibrium ensemble, it is directly seen that the variance of $\boldsymbol{E}_|$ is accordingly also the linear combination of those of the former. In other words, the fastening effect on the density fluctuations, i.e., the geometrical confinement on mass variation, presents directly in $\boldsymbol{E}_|$, and the nonlinear two-fluid dynamics tend to equi-partitionize such an effect through the thermalization process. This

Part 3:
Preliminary numerical evidence of compressibility reduction

With Yan YANG, Jun PENG and Jinxiu XU

indirectly "verifying" Zhu's (JFM2016) prediction

Helical quantum turbulence



P. CLARK DI LEONI, P. D. MININNI, AND M. E. BRACHET

PHYSICAL REVIEW A **94**, 043605 (2016)

easing the helicity constraint

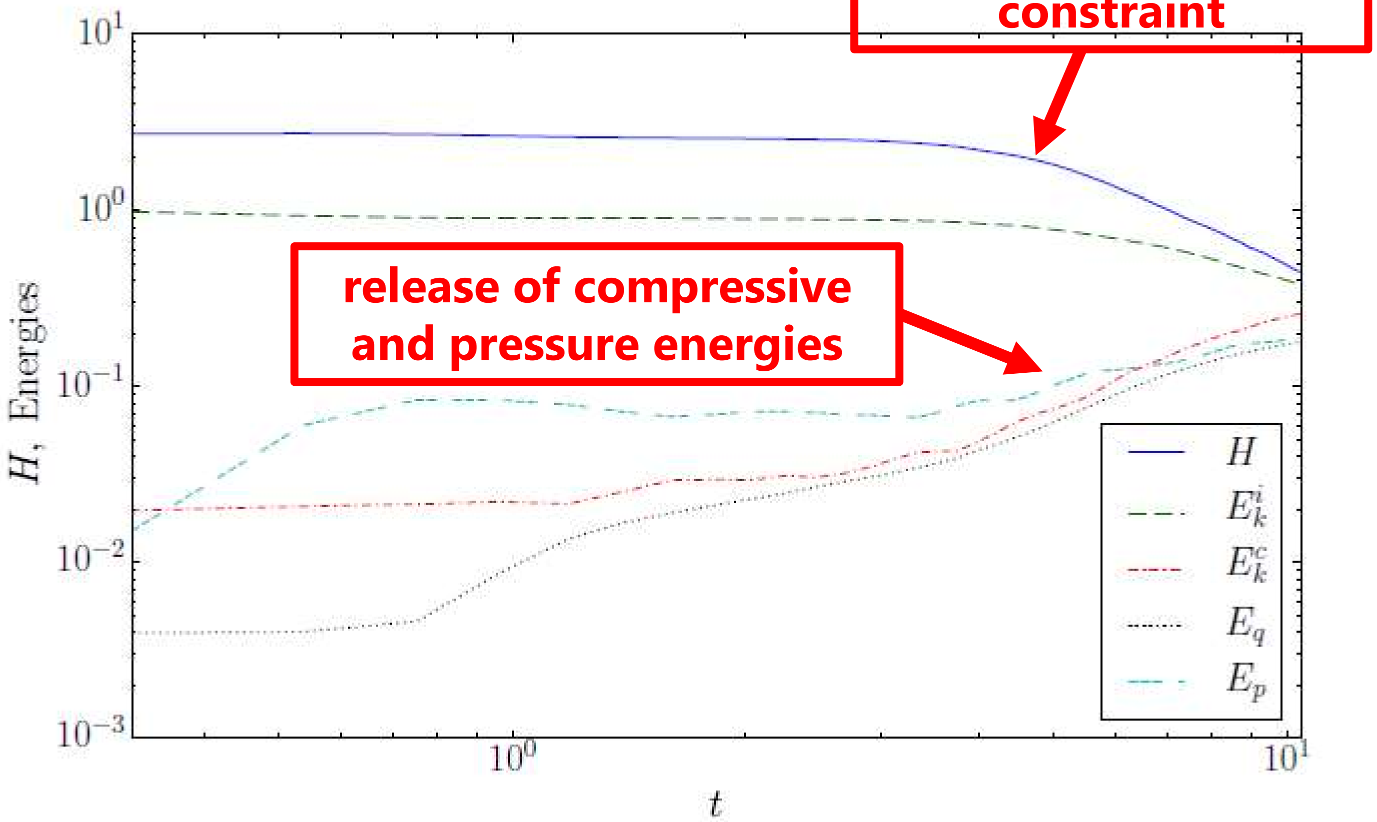


FIG. 6. Evolution of helicity and all energy components during the decay of a quantum turbulent flow with ABC initial conditions using 1024^3 grid points. Both helicity and the incompressible kinetic energy start to decay at around $t \approx 4$, in a correlated manner. The incompressible energy is redistributed into the other components, but the helicity is, in principle, dissipated.

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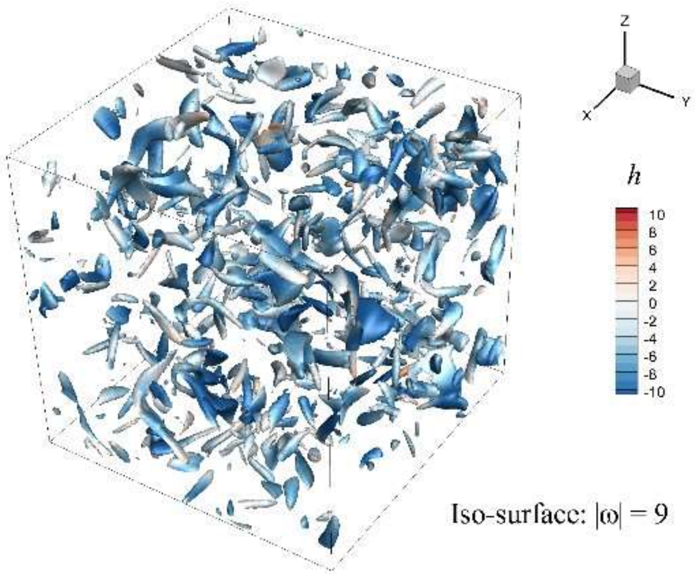
具有螺度的量子流体动力学

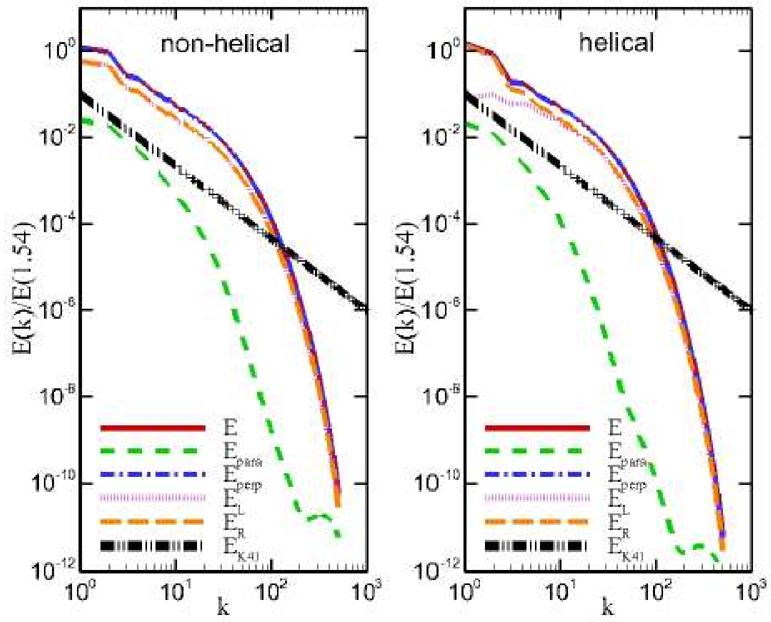
朱建州

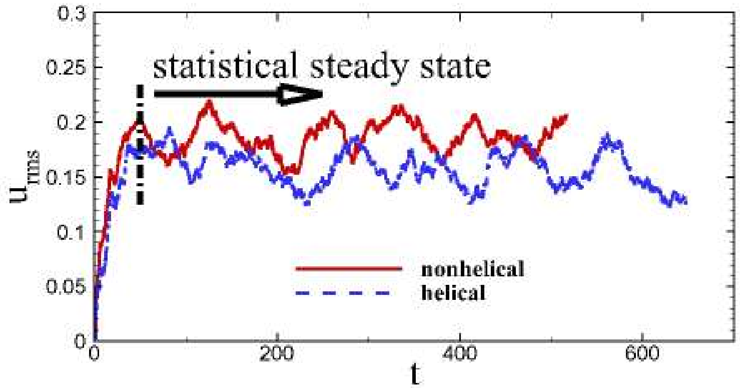
【摘要】：正Landau于1941年提出的液氦理论略带神秘色彩。它受到一些不同学者的质疑(比如,London),同时也在不同程度上得到另一种量子流体动力学表述的支持。这种不同的表述由Kronig,Thellung和Ziman等人给出,他们对经典流体动力学进行量子化,先是势流然后是由Clebsch变量表示的有旋流动。该场

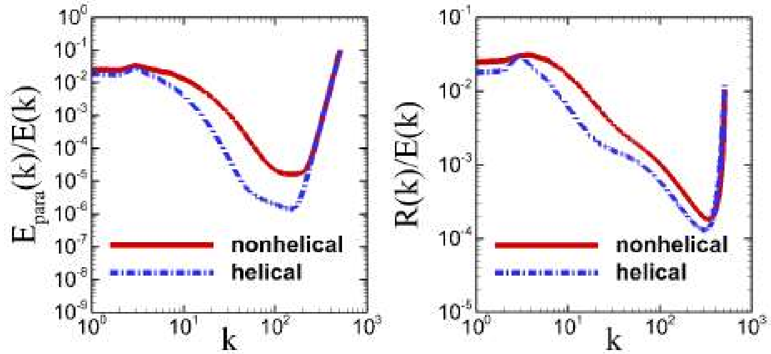
【作者单位】：南京市高淳区速诚基础与交叉科学研究中心

Y. Yang & J.-Z. Zhu, "Reducing the Noise Level by Controlling the Degree of Chirality." Chinese Conference on Computational Mechanics in conjunction with International Symposium on Computational Mechanics (CCCM-ISCM'2016). Hangzhou China, October 16-20, 2016; Y. Yang, X. Li, J.-Z. Zhu, "Helicity effect study of compressible decaying turbulence." The First aerodynamic Conference of China, Mianyang, Sichuang China (2018); J.-Z. Zhu, Y. Yang, J. Peng, "Driven (statistical steady-state) compressible helical turbulence." The First aerodynamic Conference of China, Mianyang, Sichuang, China (2018) J. Peng, J.-X. Xu, Y. Yang & J.-Z. Zhu, Helicity hardens the gas arXiv:1901.00423 [physics.flu-dyn].

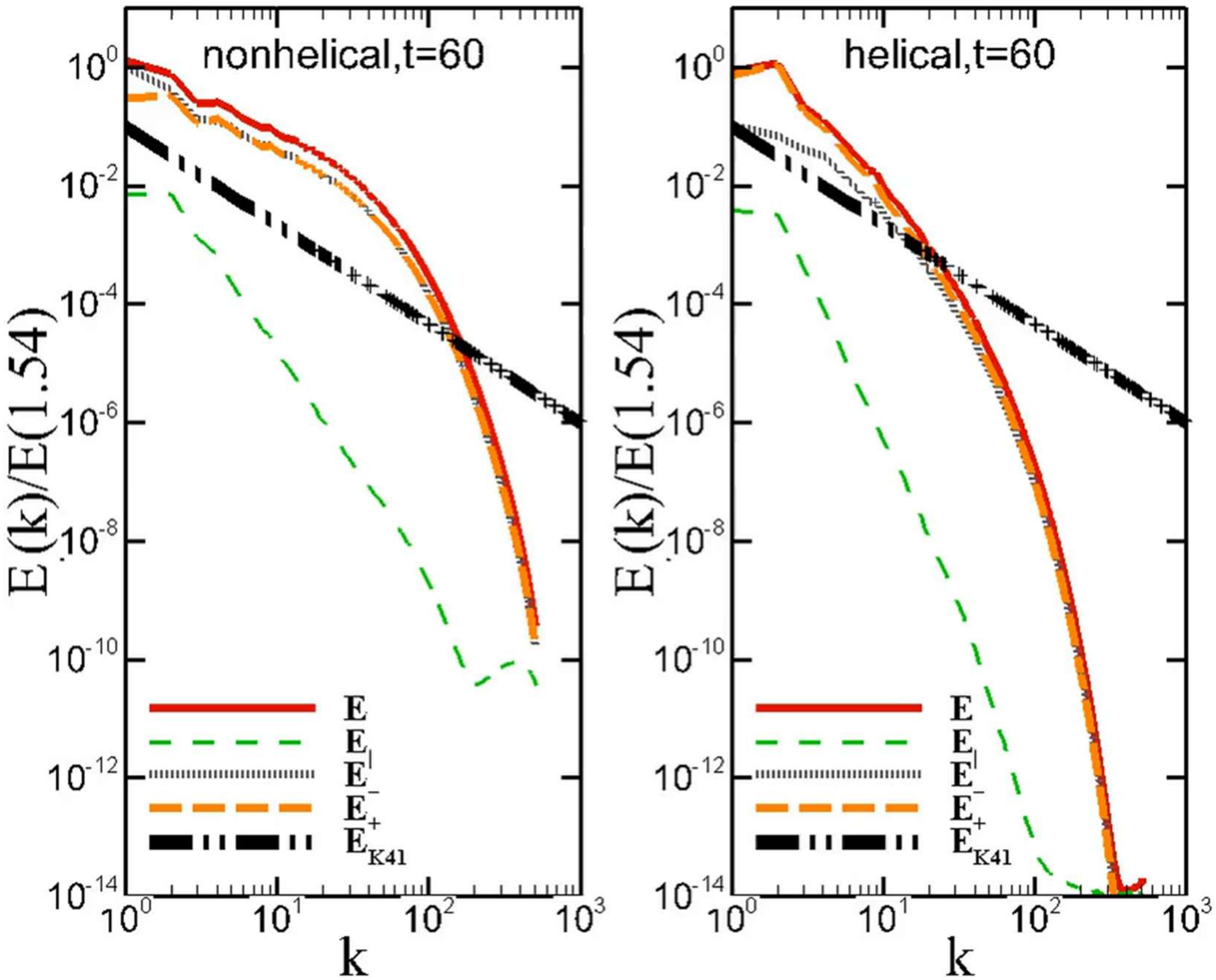


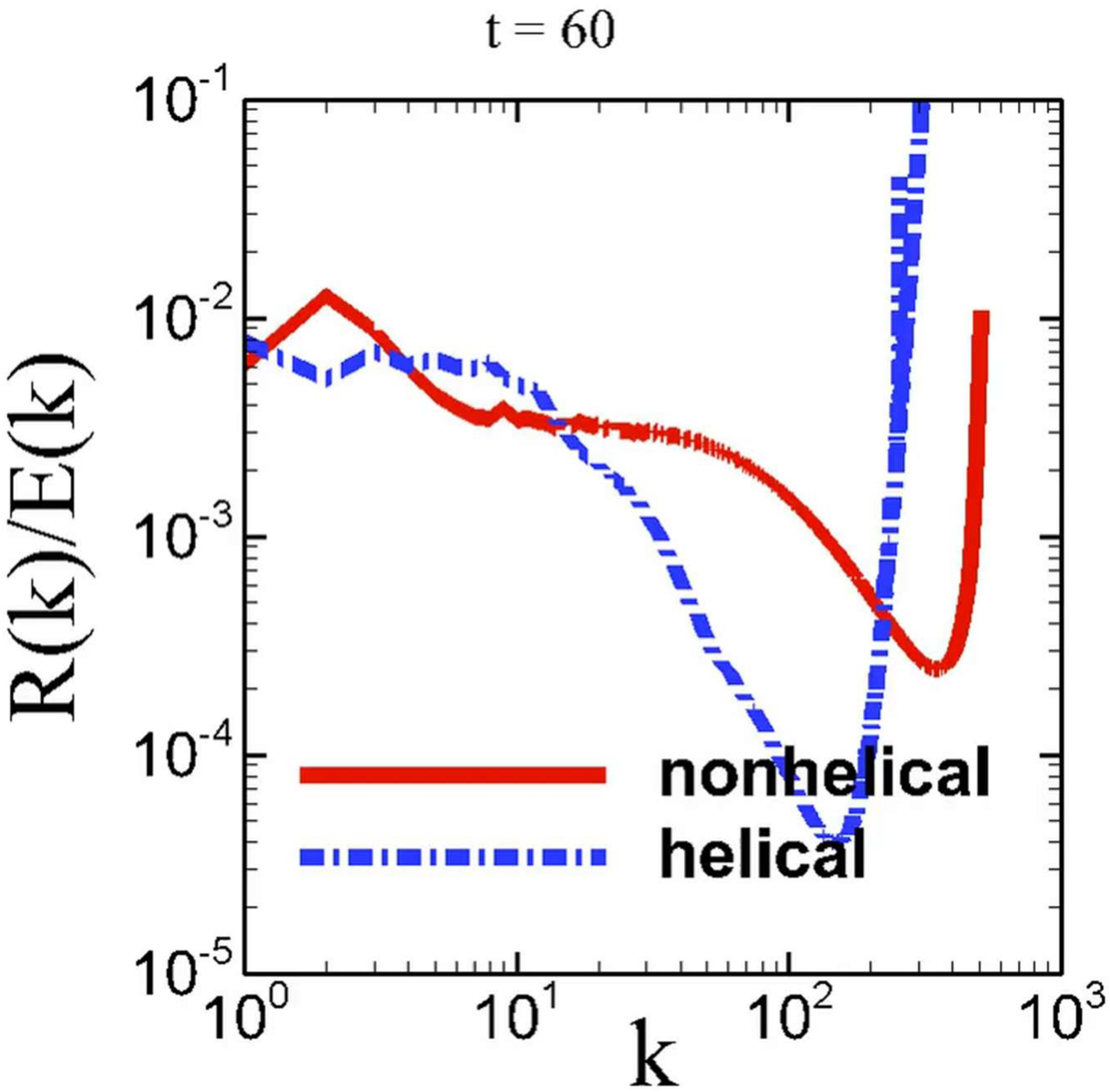

non-helical
helical
E(k)/E(1.54)
k
E
E_para
E_perp
E_L
E_R
E_K41


statistical steady state
u_rms
t
nonhelical
helical


E_para(k)/E(k)
R(k)/E(k)
k
nonhelical
helical

YANG, Y., XU, J.-X., PENG, J. & ZHU, J.-Z. 2019 Helicity hardens the gas. arXiv:1901.00423 [physics.flu-dyn].




t = 60
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
R(k)/E(k)
10^{0}
10^{1}
10^{2}
10^{3}
k
nonhelical
helical

Part 4:

Bridging CBF and turbulence:

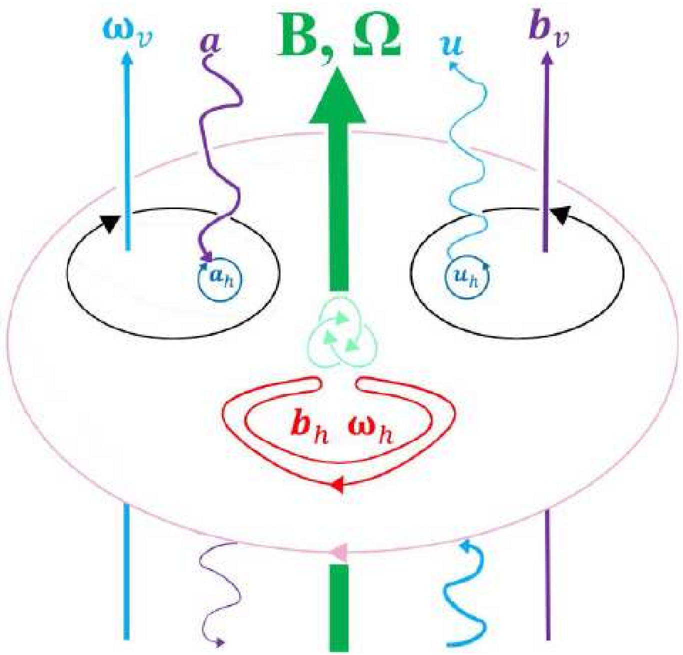
CBF-structure ensemble theory:

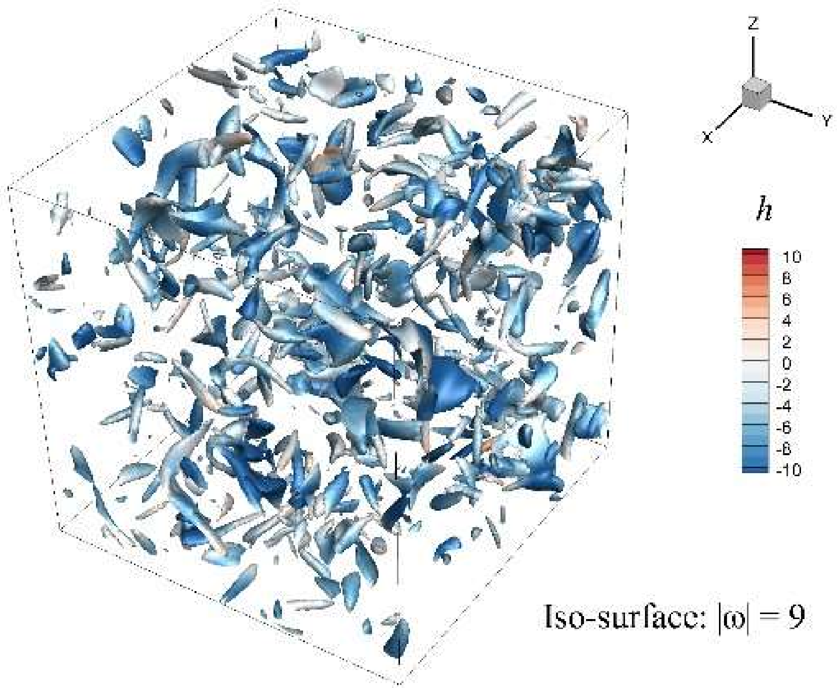
the equivalence principle for

compressible helical turbulence

（从手性基本流/场到可压缩有螺湍流的渡越）

Bridging
CBF and **turbulence**

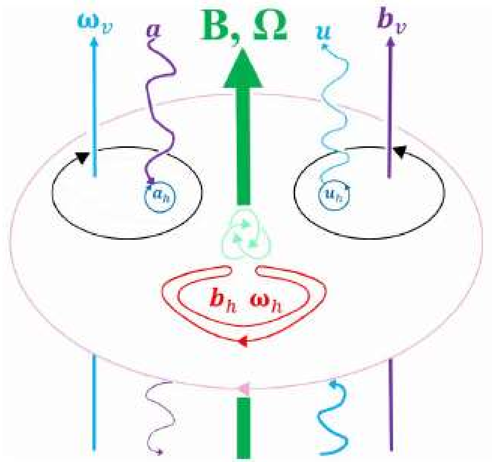


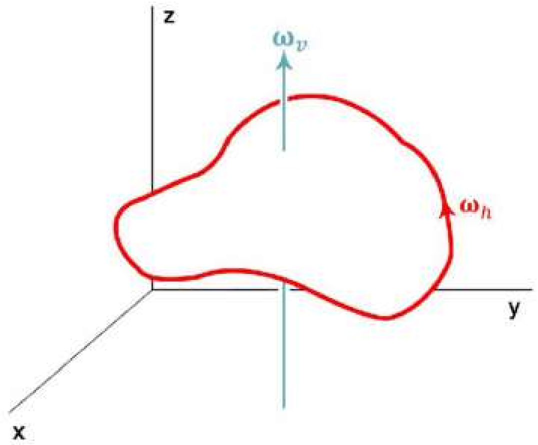


chiral/helical-base-flow/field (CBF) molecule ensemble: CBF gas kinetics

$$\nabla_{\boldsymbol{r}} \boldsymbol{u} = \begin{pmatrix} u_{x,x} & u_{y,x} & u_{z,x} \\ u_{x,y} & u_{y,y} & u_{z,y} \\ \cancel{u_{x,z}} & \cancel{u_{y,z}} & u_{z,z} \end{pmatrix}$$

$$\nabla_{\boldsymbol{r}} \boldsymbol{a} = \begin{pmatrix} a_{x,x} & a_{y,x} & a_{z,x} \\ a_{x,y} & a_{y,y} & a_{z,y} \\ \cancel{a_{x,z}} & \cancel{a_{y,z}} & a_{z,z} \end{pmatrix}$$





Equivalence principle (to be continued)

In the multi-field case, such as MHD and beyond, the gradients of different fields require their own (improper) rotations to reach the real Schur form, thus we may say that the structures at different locations $\boldsymbol{r}$ are 'similar' up to $O(3) \times O(3) \times ...$ with each rotation of each field depending on the location, a kind of 'gauge similar structure'. Extending the above HD discussions to plasma fluids, with the analogues of the TPE,[44] is straightforward, as already indicated in Fig. 1 for the case of aligned mean magnetic field $\boldsymbol{B}$ (for MHD: otherwise, for XMHD and TFPM, the mean field corresponding to the would-be ideally fronzen flux associated to the invariant generalized helicity); see Appendix A for a brief summary of the extension.

spectrum, and the turbulence to be an 'ideal (classical) gas' of such molecules. Thus, the given energy and helicity spectra of the molecular CBF determine directly those of the gas, like a real ideal gas; that is, such a turbulence model can be accurate at the level of energy and helicity. The underlying 'equivalence principle' hypothesis here is that 'gauge similarity' of local real Schur field structures can be effectively replaced by 'gauge identity' or much stronger similarity (allowing still other possible freedoms of differences that we are not interested in here) of (g)CBFs, together with the removal of the interactions.

Ensemble of **molecular** (giant/global) **(g)CBF**s for isotropic compressible helical turbulence

(too?) simple

- Global CBFs over the whole flow domain randomly distributed in an isotropic and independent way

minimal

- \`Local'-but-giant CBFs in some coarse-grained sense located randomly in space and isotropically distributed, with no/weak interactions

For self-similar (scaling-law) turbulence, the proper scale dilatation \`does not matter': statistical gauge invariance: **Schur+Weyl**

***O(3)* gauge similarity→identity or stronger similarity**

Why reduce to the mean rotation instead of the mean velocity?

Mean velocity

- Galilean invariance, thus trivial with no consequence

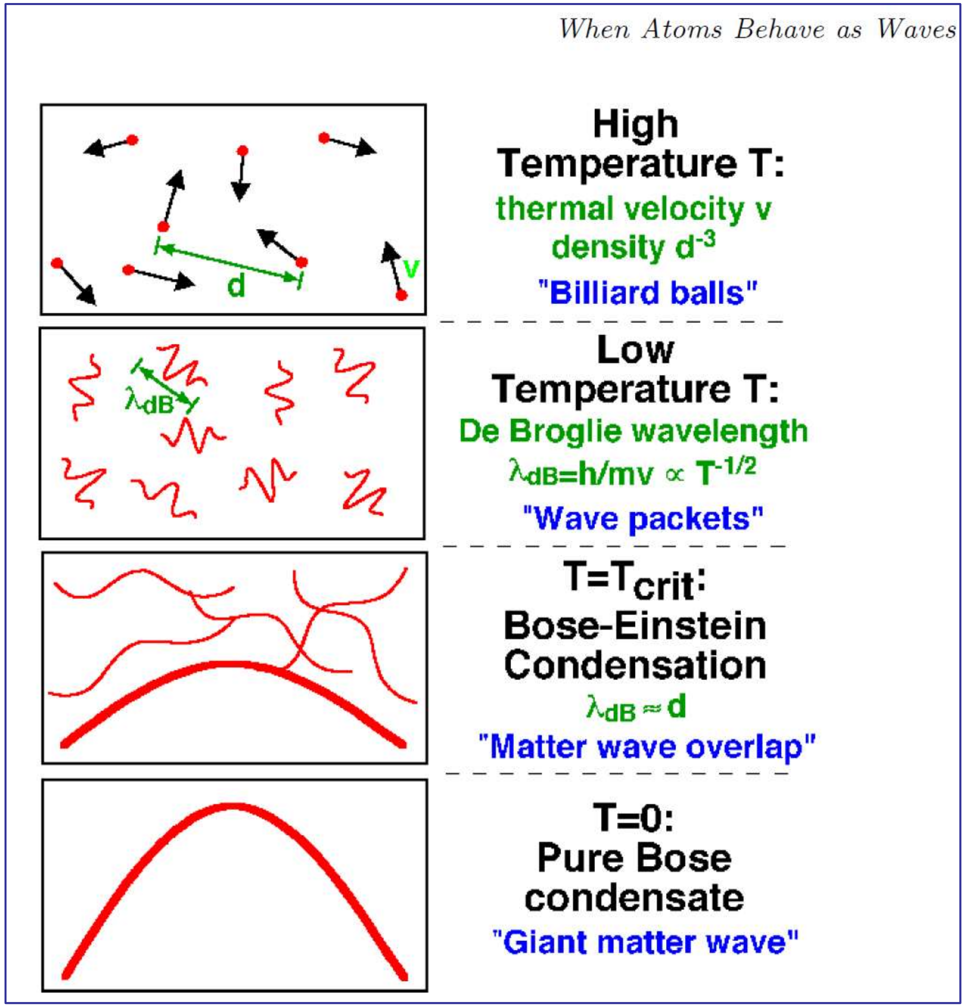
Mean rotation

- Taylor-Proudman effect, thus nontrivial with fastening consequence

Akin to Kraichnan's (1964)
random Galilean transformation

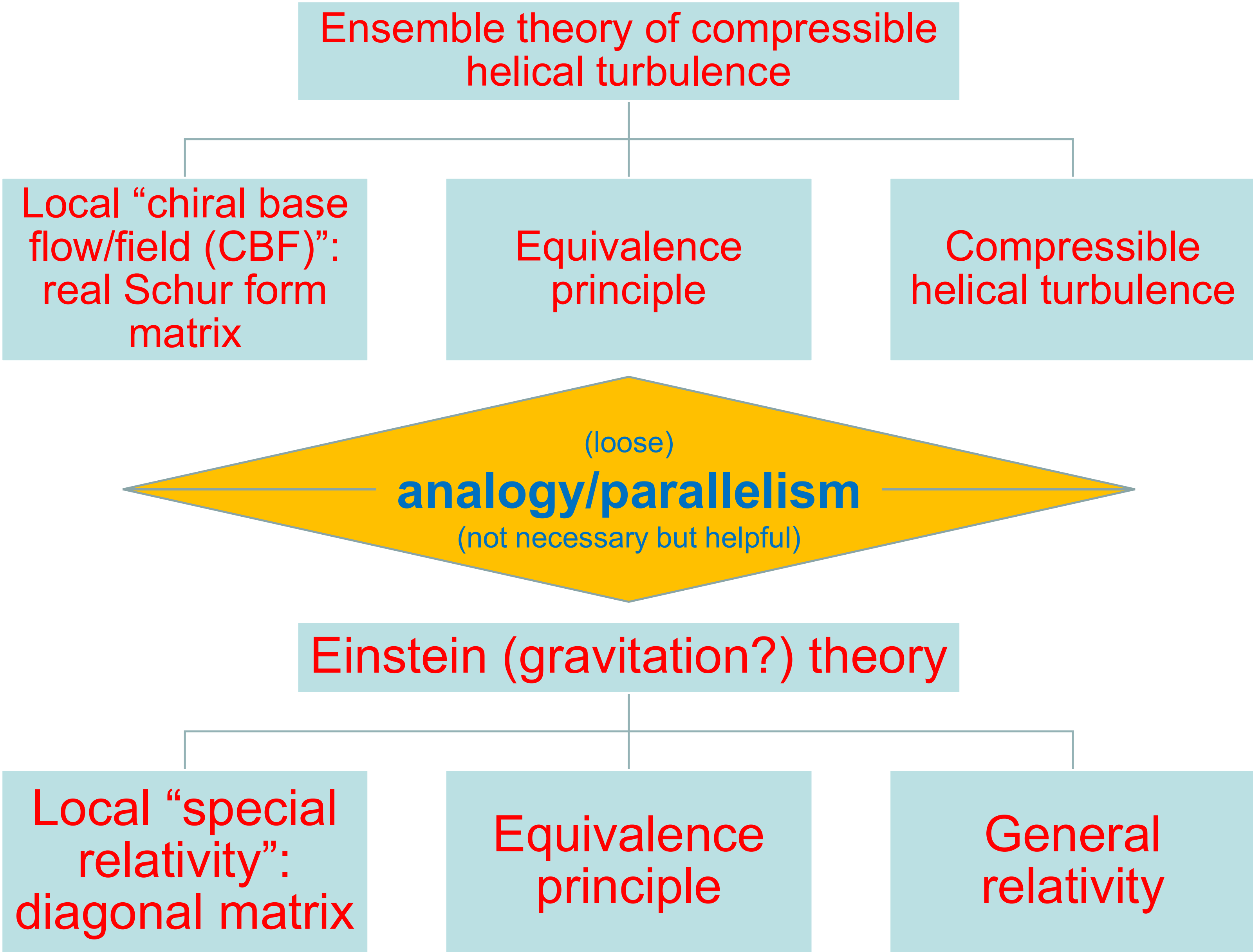
Another loose analogy:

gCBF and gMW: just for motivating possible physical pictures



CBFs of positive cosines of their vorticity angles ('similar') but with opposite helicities may attract each other (reminiscent of the Cooper pair in the BCS theory of superconductivity) to form (local) gCBFs of helicity H with no or weak interactions

And, more details, **Galilean and Lorentz invariances, Taylor-Proudman fastening and Einstein length-contraction effects ……, in the big analogy:**

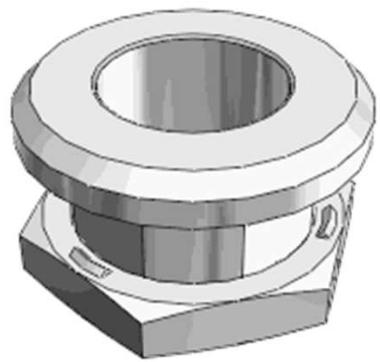


Part 5:
Possible applications

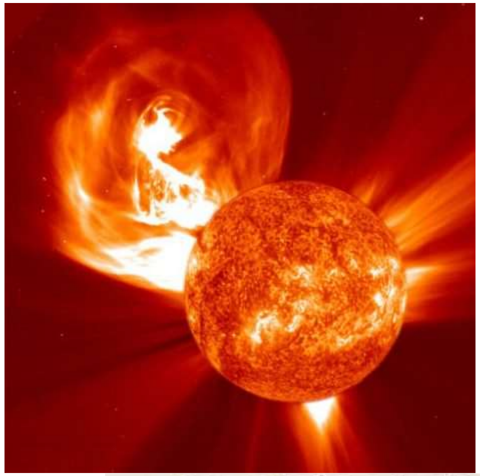
“fasten” notion “扎紧”之概念








“扎紧” 者，出于生产之用，入乎食色之需，乃日常观念也



“螺控声”理论技术

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“Unlike other organisms and viruses, coronavirus has RNA as their genetic material --- a very rare form of the virus. It’s an RNA virus. The ribonucleic acid of it is surrounded by the nucleocapsid. The capsid of coronavirus is helical in symmetry.”

Acknowledgements:

www.SCCFIS.org

NSFC

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Statement: figures in Page 3 and 38 are taken from WWW.