Online Advanced Study Program on Helicities in Astrophysics and Beyond
(Helicity 2020-2021)

# Dissipation-Range Nonuniversality from Helicity Fastening Effects on the Complex Singularities

扎紧湍流：
螺度效应导致耗散区非普适性的复奇点视角

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2021年5月21日

# Outline

- 1. Brief review of the advancements :
- 长报告可见于：https://helicity2020.izmiran.ru/rec/zhu.mp4;
- 最新的理论文章： Phys. Plasmas 28, 032302 (2021); doi: 10.1063/5.0031108.
- Motivation: “strong” equivalence principle(s) and application(s): needing more detailed knowledge and deeper understanding

- 2. Theoretical consideration with complex singularities

- 3. Looking closer at our numerical data: Nonuniversality and universality in the dissipation-range: Factorization

- 4. (Chiral)-base-flow/(helical-)real-Schur-flow complex singularities: numerical observations and the programme of “strong” principle of equivalence
- 最新的一些数值模拟视频：https://mp.weixin.qq.com/s/miUUOo342LcGLgIoO1-NJw

# Taylor-Proudman fastening effect in chiral/helical base flows

## 扎紧效应之流体力学/几何、弱等效原理

$$\nabla_{\boldsymbol{r}} \boldsymbol{u} = \begin{pmatrix} u_{x,x} & u_{y,x} & u_{z,x} \\ u_{x,y} & u_{y,y} & u_{z,y} \\ \cancel{u_{x,z}} & \cancel{u_{y,z}} & u_{z,z} \end{pmatrix}$$

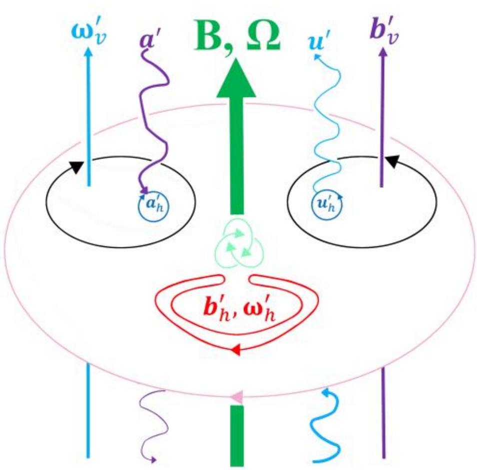
**Real Schur Form (RSF)**

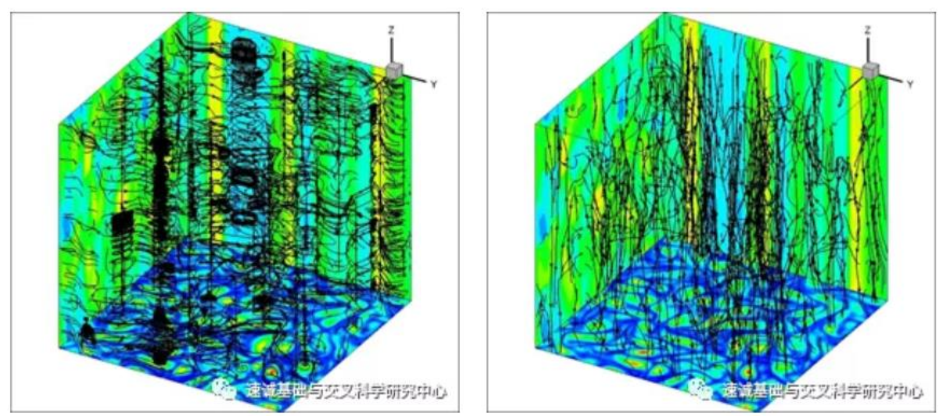
[c.f., 1. J.-Z. Zhu, Phys. Fluids **30**, 031703 (2018);

[5]C. J. Keylock and S. Tian *et al.* have applied RSF in their respective studies of flow structures (private communications, 2017 and 2018).

[6]The local RSF could be helpful for clarifying the *global regularity* issue. And, this is reminiscent of the locally inertial coordinates, supporting the *principle of equivalence* in general relativity. We can also work with other vector gradients, whose antisymmetric parts correspond to the magnetic field, the electric current or else, and may formulate a kind of principle of equivalence for the specific dynamics.

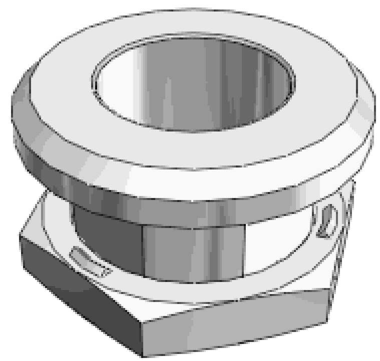
2. Phys. Plasmas **28**, 032302 (2021): weak equivalence principle]



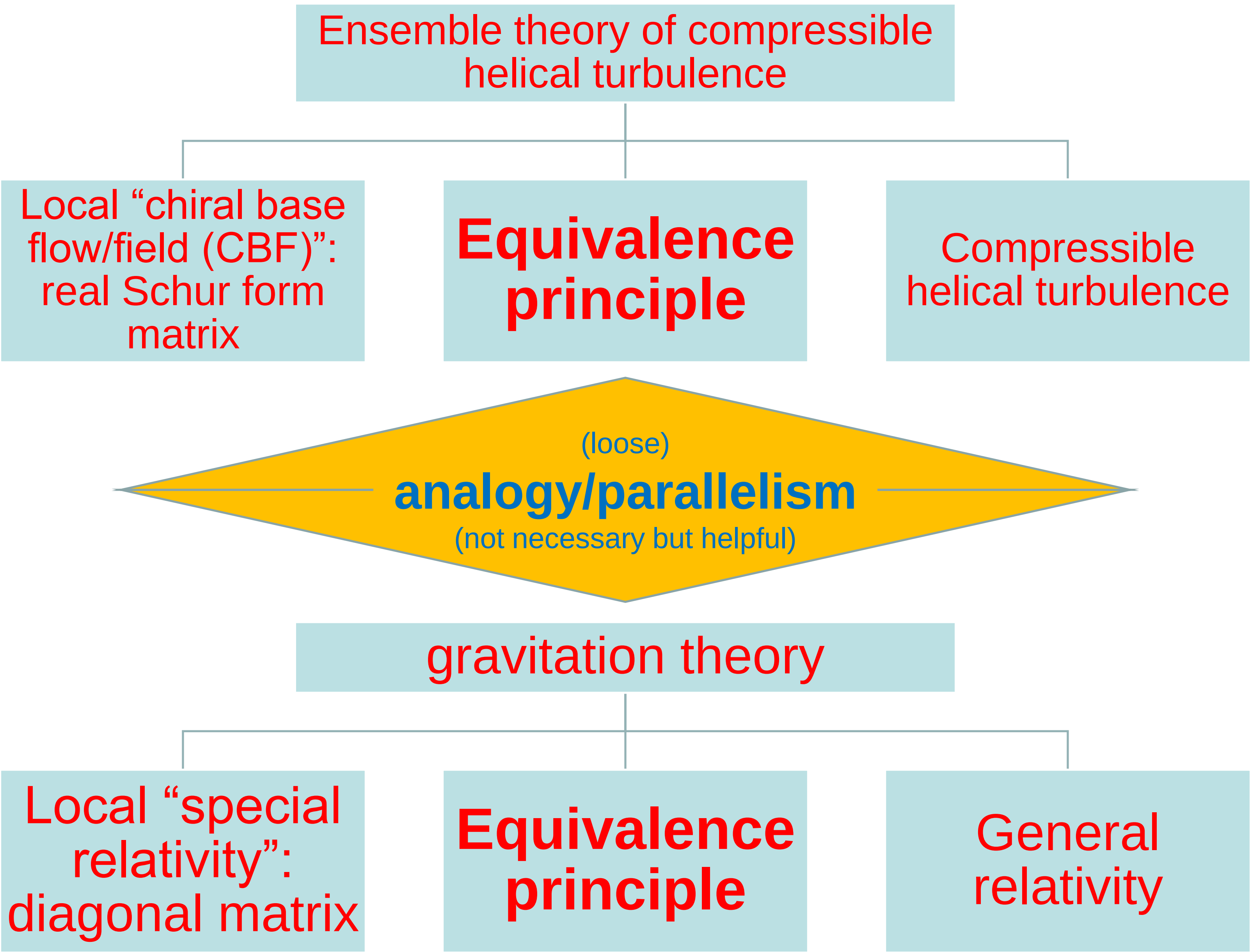
温度（左切面）、密度（右切面）、涡强度（底切面）和速度流线；右：温度（左切面）、密度（右切面）、涡强度（底切面）和涡矢量流线

# 扎紧之概念 The notion of fastening



夫“扎紧” 者，出于生产之用，入乎食色之需，乃日常观念也

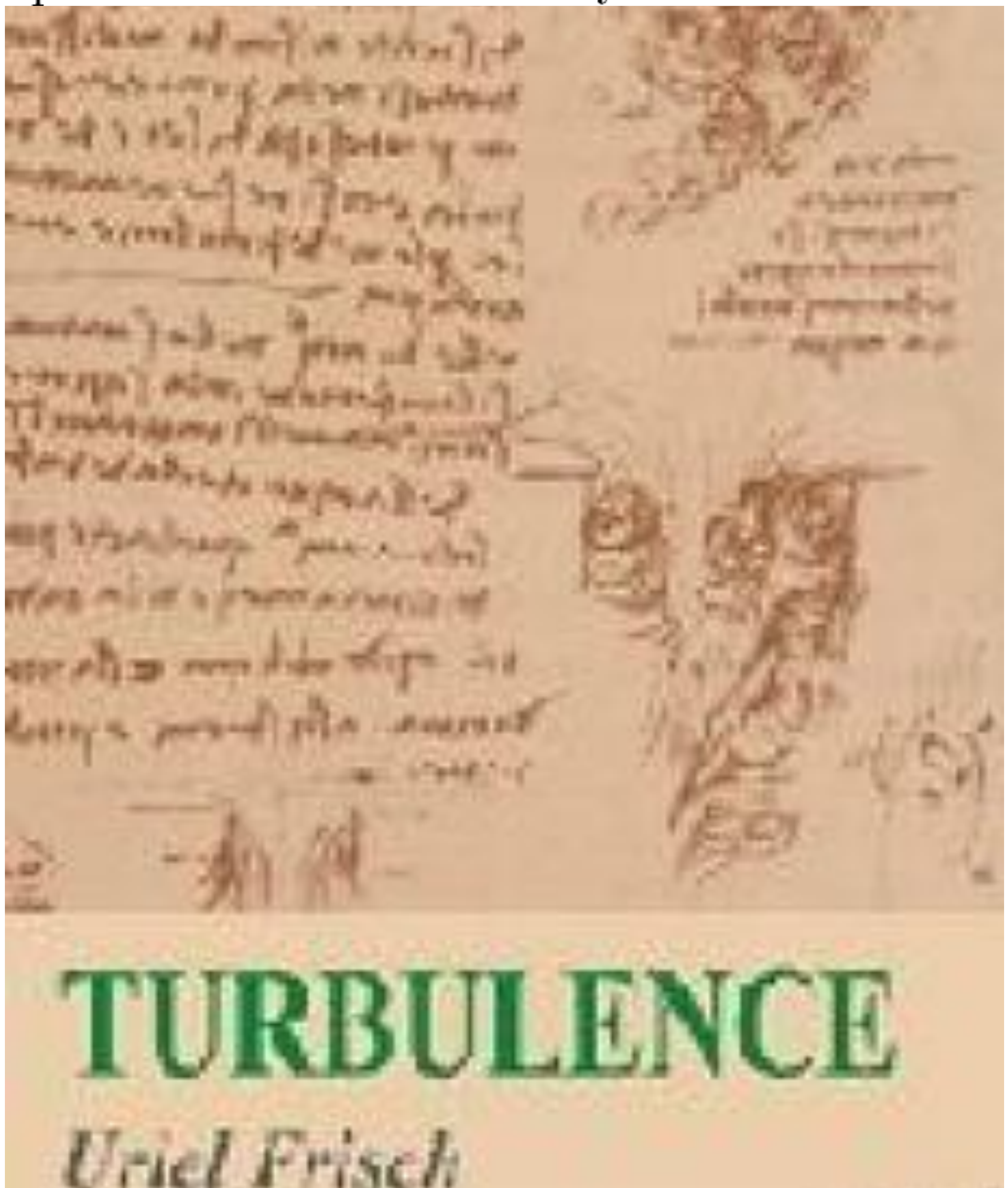
# **Strong equivalence principle?**: needing more detailed knowledge and deeper understanding



# theoretical considerations with complex singularities

**The first hypothesis of similarity.** *For the locally isotropic turbulence the distributions* $F_n$ *are uniquely determined by the quantities* $\nu$ *and* $\bar{\epsilon}$.

† First published in Russian in *Dokl. Akad. Nauk SSSR* (1941) **30**(4). Paper received 28 December 1940. This translation by V. Levin, reprinted here with emendations by the editors of this volume.




TURBULENCE
Uriel Frisch

Kolmogorov's (1941a) original derivation of the dissipation scale $\eta$ was rather different:

**Kolmogorov's first universality assumption.**[14] *At very high, but not infinite Reynolds numbers, all the small-scale statistical properties are uniquely and universally determined by the scale $\ell$, the mean energy dissipation rate $\varepsilon$ and the viscosity $\nu$ (or, equivalently, by $\ell$, $\varepsilon$ and $\eta$).*

'Small-scale' is here understood as scales small compared to the integral scale, i.e. inertial-range and dissipation-range scales. By a simple dimensional argument, the first universality assumption implies the following universal form for the energy spectrum at large wavenumbers

$$E(k) = \varepsilon^{2/3} k^{-5/3} F(\eta k), \tag{6.69}$$

where $F(\cdot)$ is a universal dimensionless function of a dimensionless argument. By the second universality assumption of Kolmogorov (Section 6.1), $F(\cdot)$ tends to a finite positive limit (the Kolmogorov constant) for vanishing argument. The universality of the whole function $F(\cdot)$ has been questioned by Frisch and Morf (1981), using the same sort of argument that Landau developed for the Kolmogorov constant (see Section 6.4).

There were several early attempts to determine the functional form of $F(\cdot)$ at high wavenumbers. They will not be reviewed here (see, e.g., Monin and Yaglom 1975). The most interesting remark was made by von Neumann (1949). He observed that an analytic function has a Fourier transform which falls off exponentially at high wavenumbers. The logarithmic decrement is equal to the modulus $\delta$ of the imaginary part of the position of the singularity in complex space nearest to the real domain. Therefore, in von Neumann's view, exponential fall-off at high $k$ was more likely than the rapid algebraic fall-off proposed by Heisenberg (1948). Actually, for a random homogeneous function, the situation is a bit more complicated: there is a probability distribution $P(\delta)$ and thus the form for the energy spectrum at high $k$ is the Laplace transform of $P(\delta)$ near its minimum value $\delta_\star$ (Frisch and Morf 1981).

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# No-go universal equilibrium theory?

## [from Kolmogorov to Landau and to ……]

Publications recently led by my Chinese seniors, e.g.:

**UNIVERSAL-EQUILIBRIUM-RANGE SPECTRUM OF TURBULENCE**

J. Qian Gao Zhi

(*Institute of Mechanics, Academia Sinica*)

按照 Kolmogorov[1] 1941 年提出的理论，完全发展湍流的普适平衡区的能谱 $E(k)$ 由运动粘度 $\nu$ 和能量耗散率 $\varepsilon$ 所决定，而且[2-4]

$$E(k) = \varepsilon^{2/3} k^{-5/3} F(k/k_d), \tag{1}$$

$$F(x) = 1.19(1 + 5.3x^{\frac{2}{3}})\exp(-5.4x^{4/3}) \tag{4}$$

$$F(x) = 1.19(1 + 19.4x)\exp(-6.1x) \tag{15}$$

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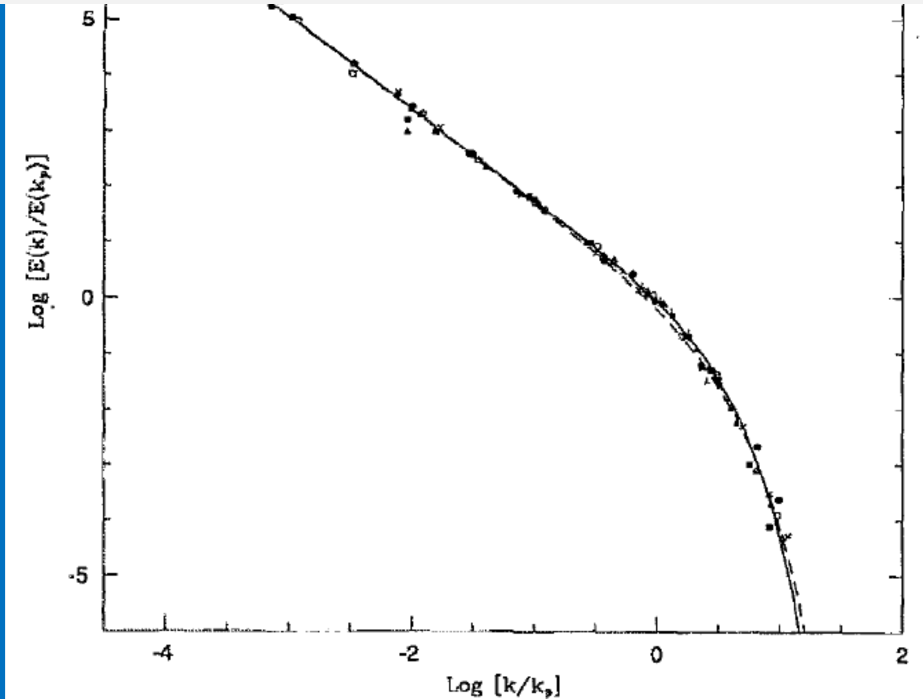
**On the universal form of energy spectra in fully developed turbulence**

Zhen-Su She
*Department of Mathematics, University of Arizona, Tucson, Arizona 85721*

Eric Jackson
*Program in Applied & Computational Mathematics, Princeton University, Princeton, New Jersey 08544*

$$E(k)=E(k_p)\left[\left(\frac{k}{k_p}\right)^{-5/3}+\alpha\left(\frac{k}{k_p}\right)^{-\beta}\right]e^{-\mu k/k_p}, \qquad (4)$$



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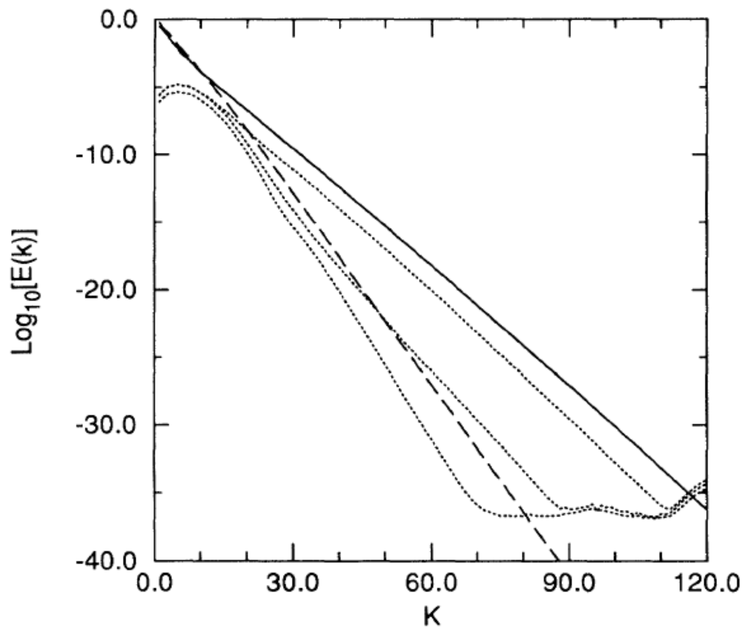
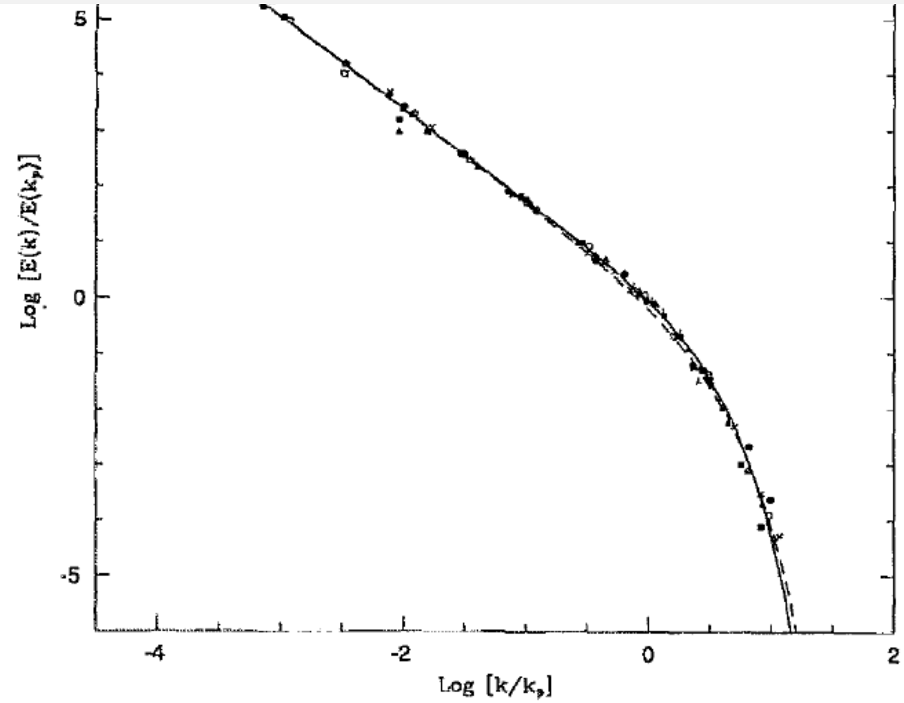



FIG. 1. Linear-log plot of $E(k)$ vs $k$. Solid line, DNS spectrum; dotted lines, subregional spectra with largest, median, and smallest values of $E(k=60)$; dashed line, DIA spectrum.




**Far-Dissipation Range of Turbulence**

Shiyi Chen and Gary Doolen

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*ied and Computational Mathematics, Princeton University, Pri*

Zhen Su She

*Mathematics Department, University of Arizona, Tucson, Arizo*

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[from **Kolmogorov** to **Landau** and to… *cont'd*]

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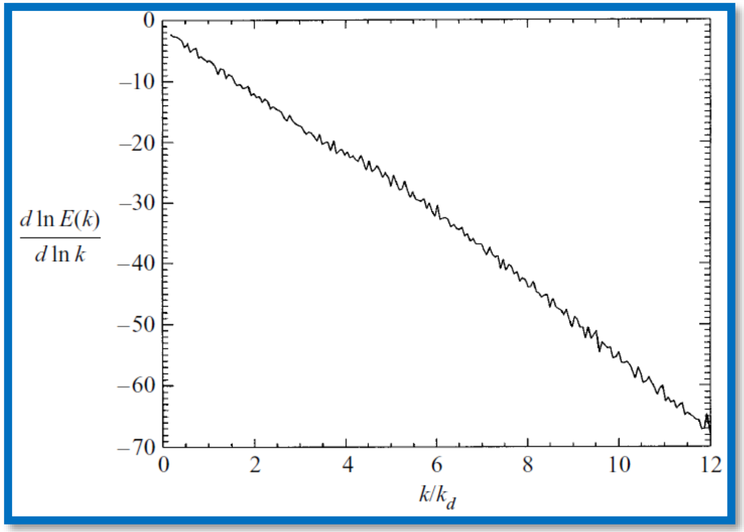

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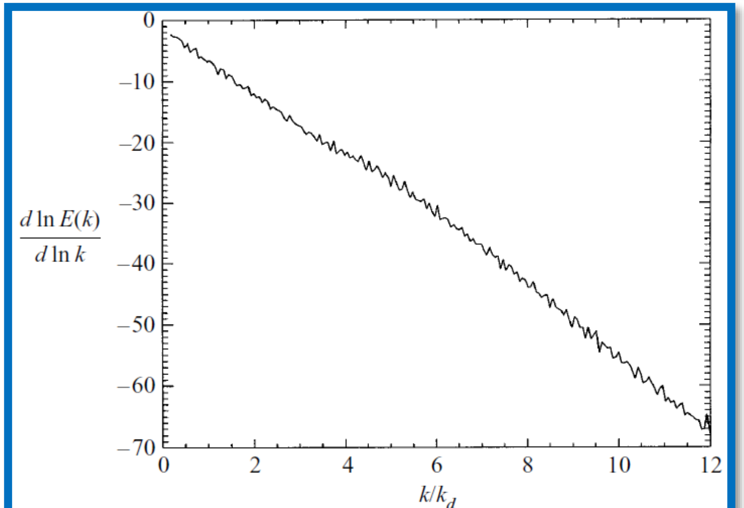


**Analysis of the dissipative range of the energy spectrum in grid turbulence and in direct numerical simulations**

Anastasiia Gorbunova,[1,2] Guillaume Balarac,[2,4] Mickaël Bourgoin,[3] Léonie Canet,[1,4] Nicolas Mordant,[2] and Vincent Rossetto[1]

$$C(t, \vec{k}) = A\epsilon^{2/3}k^{-11/3}\exp[-\gamma(\epsilon L)^{2/3}(tk)^2 + O(k)],$$

scaling variable. Indeed, dimensional analysis (Kolmogorov theory) predicts a dynamical critical exponent $z = 2/3$ and thus a scaling variable $tk^z = tk^{2/3}$. The dependence in $tk$ is thus a breaking

where $\gamma$ and $A$ are nonuniversal constants [31], $\epsilon$ is the mean energy dissipation rate, and $L$ is the integral scale. The leading term in $k^2$ in the exponential is exact, whereas the factor in front of the exponential is not. Indeed, the exponent of the power law could be modified by terms of order $\ln(k)$ in the exponential, entering in the indicated $O(k)$ corrections. The leading behavior of the correlation

$$E(k) = A'\epsilon^{2/3}(k\eta)^{-\beta}\exp[-\mu(k\eta)^{\alpha}], \quad (6)$$

where $\beta = 5/3$, $\alpha = 2/3$, and $\mu$ and $A'$ are positive nonuniversal constants related to $\gamma$ and $A$, respectively. This behavior is valid at large wave numbers that are still controlled by the fixed point, which should correspond to the NDR. Indeed, in the FDR, for very small spatial scales, the spectrum should be regularized by the viscosity and should be analytical in real space, which means that it should decay as a pure exponential in $k$ space. The unusual emergence of a stretched

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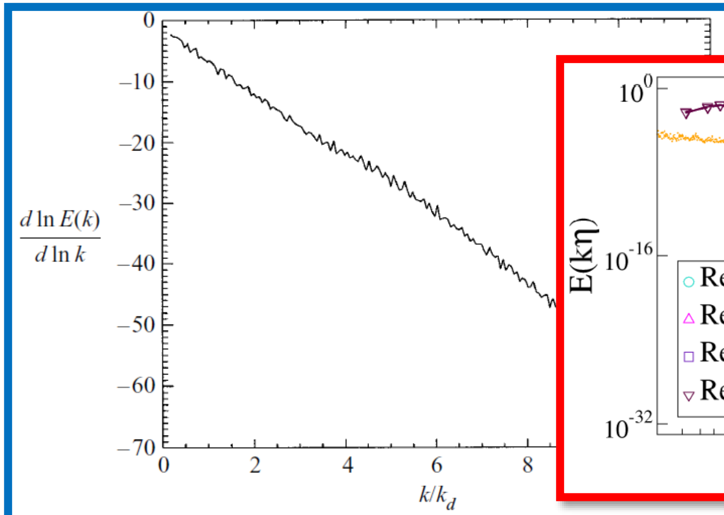


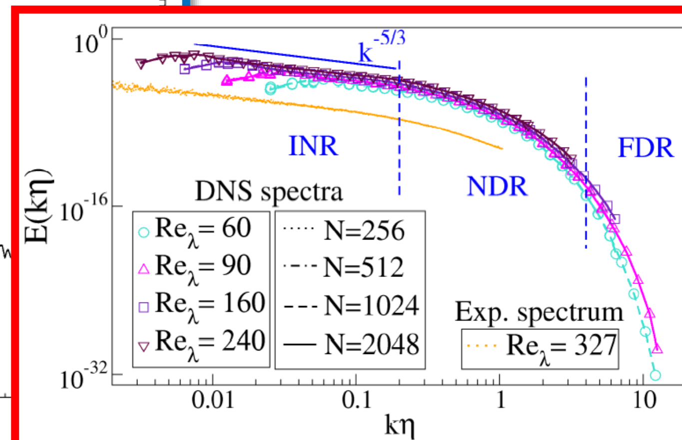


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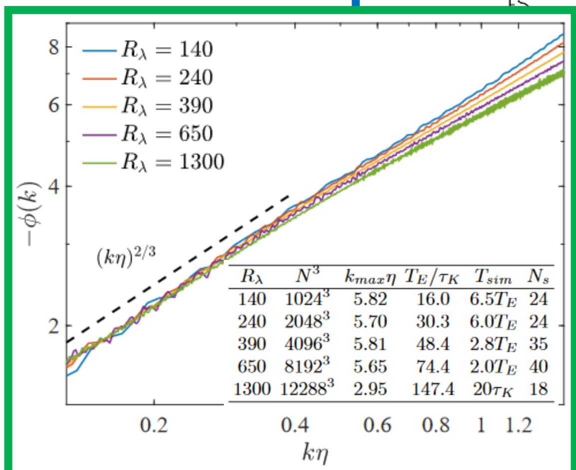


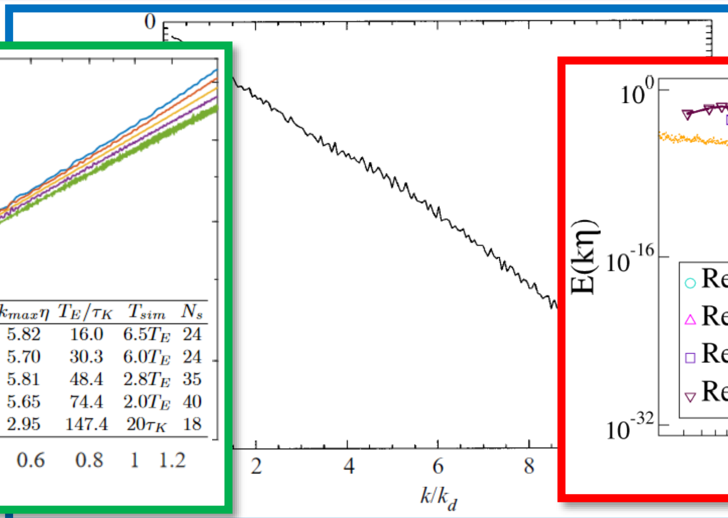


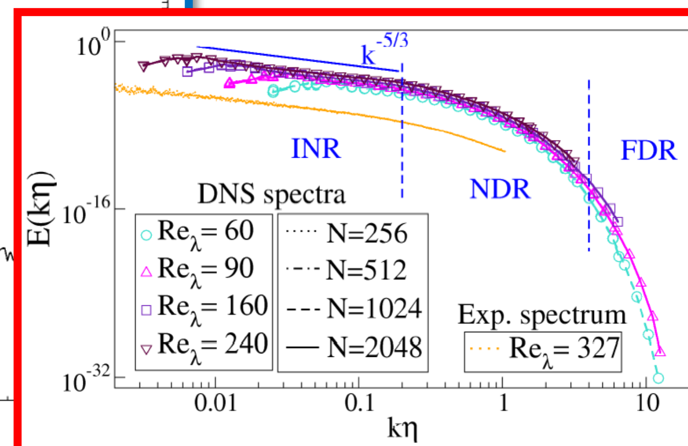


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where $\beta = 5/3$, $\alpha = 2/3$, and $\mu$ and $A'$ are positive nonuniversal constants related to $\gamma$ and $A$, respectively. This behavior is valid at large wave numbers that are still controlled by the fixed point, which should correspond to the NDR. Indeed, in the FDR, for very small spatial scales, the spectrum should be regularized by the viscosity and should be analytical in real space, which means that it should decay as a pure exponential in $k$ space. The unusual emergence of a stretched

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PHYSICAL REVIEW A VOLUME 23, NUMBER 5 MAY 1981

## Intermittency in nonlinear dynamics and singularities at complex times

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*Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138*
*and Centre National de la Recherche Scientifique, Observatoire de Nice, BP 252, 06007 Nice-Cedex, France**

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(Received 30 June 1980)

High-pass filtering of turbulent velocity signals is known to produce intermittent bursts. This is, as shown, a general property of dynamical systems governed by nonlinear equations with band-limited random forces or intrinsic stochasticity. It is shown that singularities for complex times determine the very-high-frequency behavior of the solution and show up in the high-pass filtered signal as bursts centered at the real part of the singularity and with overall amplitude decreasing exponentially with the imaginary part. Near a singularity, nonlinear interactions, however weak they may be on the real axis, acquire unbounded strength. Investigations of singularities by nonperturbative methods is thus essential for quantitative analysis of high-frequency or high-wave-number properties. In contrast to results based on two-point closures, the high-frequency dissipation-range spectrum is actually not universal with respect to the low-frequency forcing. Unlimited intermittency is demonstrated, i.e., the flatness of the high-pass filtered solution grows indefinitely with filter frequency. This gives strong support to a conjecture of Kraichnan [Phys. Fluids **10**, 2080 (1967)] about intermittency in the dissipation range of turbulent flows. The analysis is carried out in great detail for the nonlinear Langevin equation $m\dot{v} = -\gamma v - v^3 + f(t)$. Lorenz's three mode system and Burgers's model are also discussed. Conjectures are made about Navier-Stokes turbulence which can be checked experimentally and numerically.

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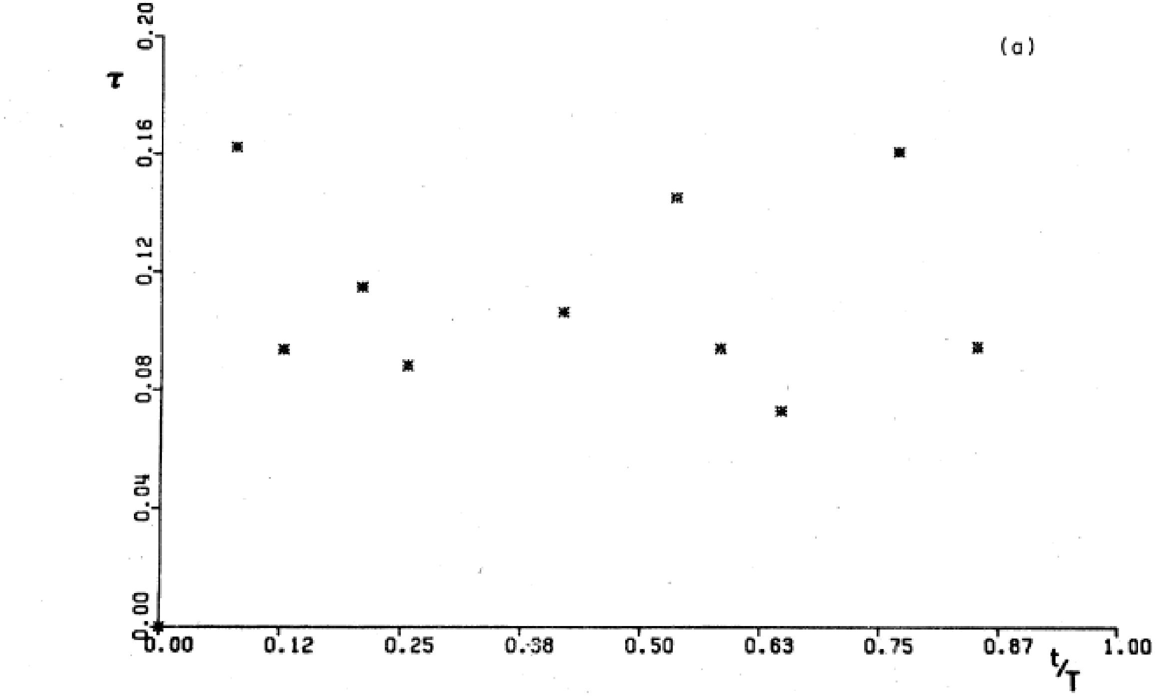
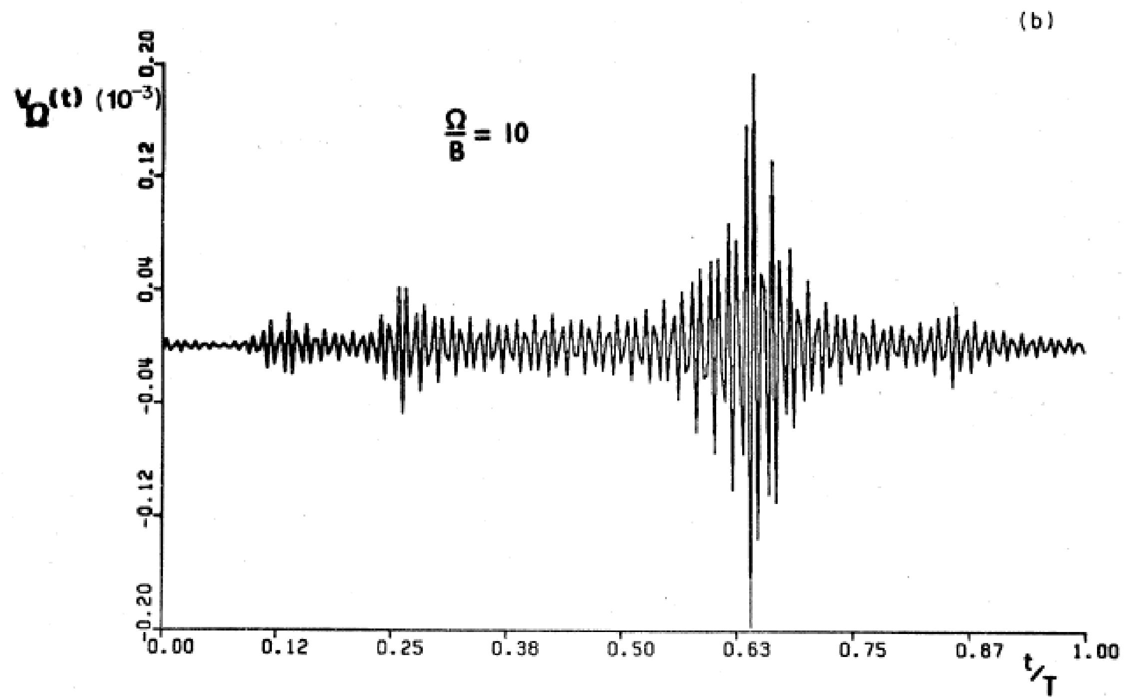

(a)
τ
0.20
0.16
0.12
0.08
0.04
0.00
0.00
0.12
0.25
0.38
0.50
0.63
0.75
0.87
1.00
t/T


(b)
$V_\Omega(t)$ $(10^{-3})$
$\frac{\Omega}{B} = 10$
0.20
0.12
0.04
-0.04
-0.12
-0.20
0.00
0.12
0.25
0.38
0.50
0.63
0.75
0.87
1.00
t/T

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**how helicity influences the overall nature and/or distribution of them, and then the flow compressibility**.

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**Ansatz**: $I(k)$ is a power function (in the interested regime).

Some considerations partly related to complex singularities from, e.g., SulemSulemFrisch1983jcp:

Consider an analytic function $v(z)$ with singularities at complex location $z_j$, in the neighborhood of which it behaves as

$$v(z) \sim (z - z_j)^{\mu_j} \tag{2.1}$$

($\mu_j$ is assumed not to be a positive integer). The behavior of the Fourier transform for $k \to +\infty$ is governed by the singularity of the upper half-space closest to the real domain that is not a multiple pole; if this singularity is located at $z_* = x_* + i\delta$ and has an exponent $\mu$, one has

$$\hat{v}_k \sim |k|^{-(\mu+1)} e^{-k\delta} e^{ix_* k}, \qquad k \to +\infty. \tag{2.2}$$

A derivation of this property can be found in [7, p. 255]. It requires that $v(z)$ be growing not faster than an exponential as $|z| \to \infty$ and that the singularities be isolated points. Note that when several singularities are relevant asymptotically, $|\hat{v}_k|$ may display an oscillatory behavior.

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Underlain by the “coherent” structure of the complex singularities of different variables: think about a shock

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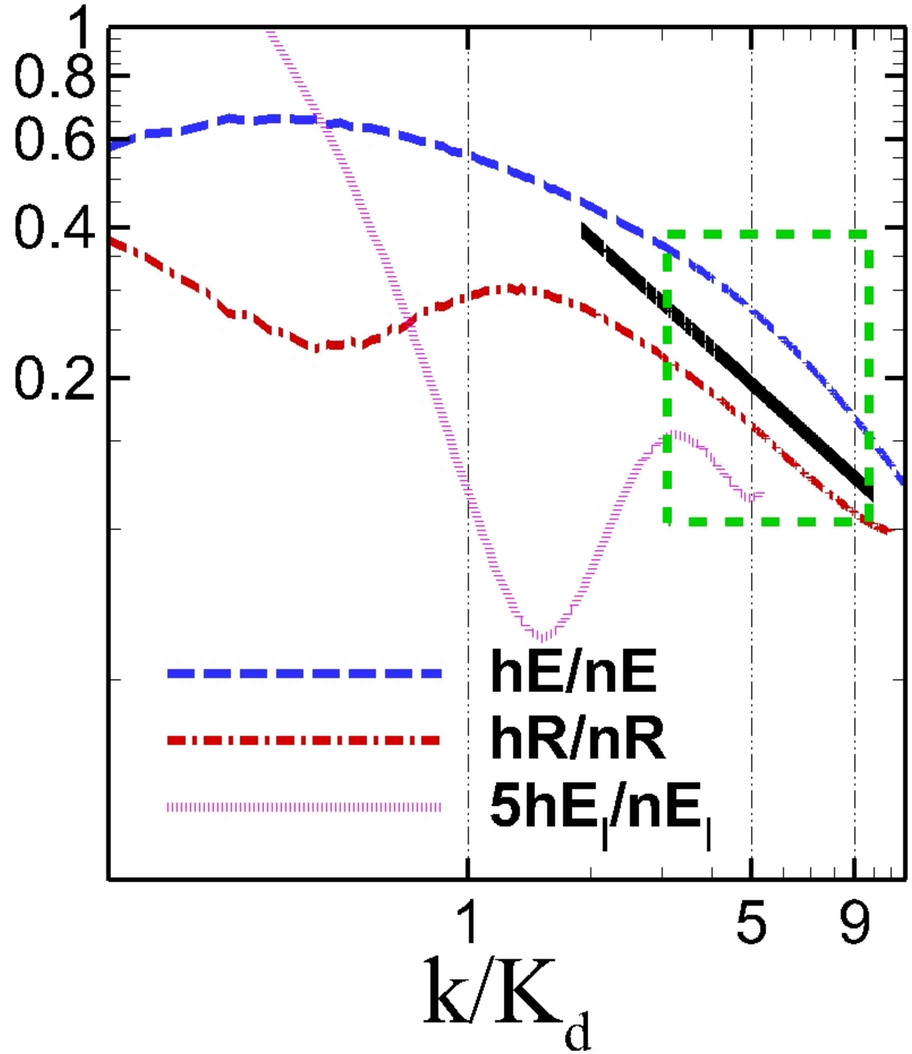


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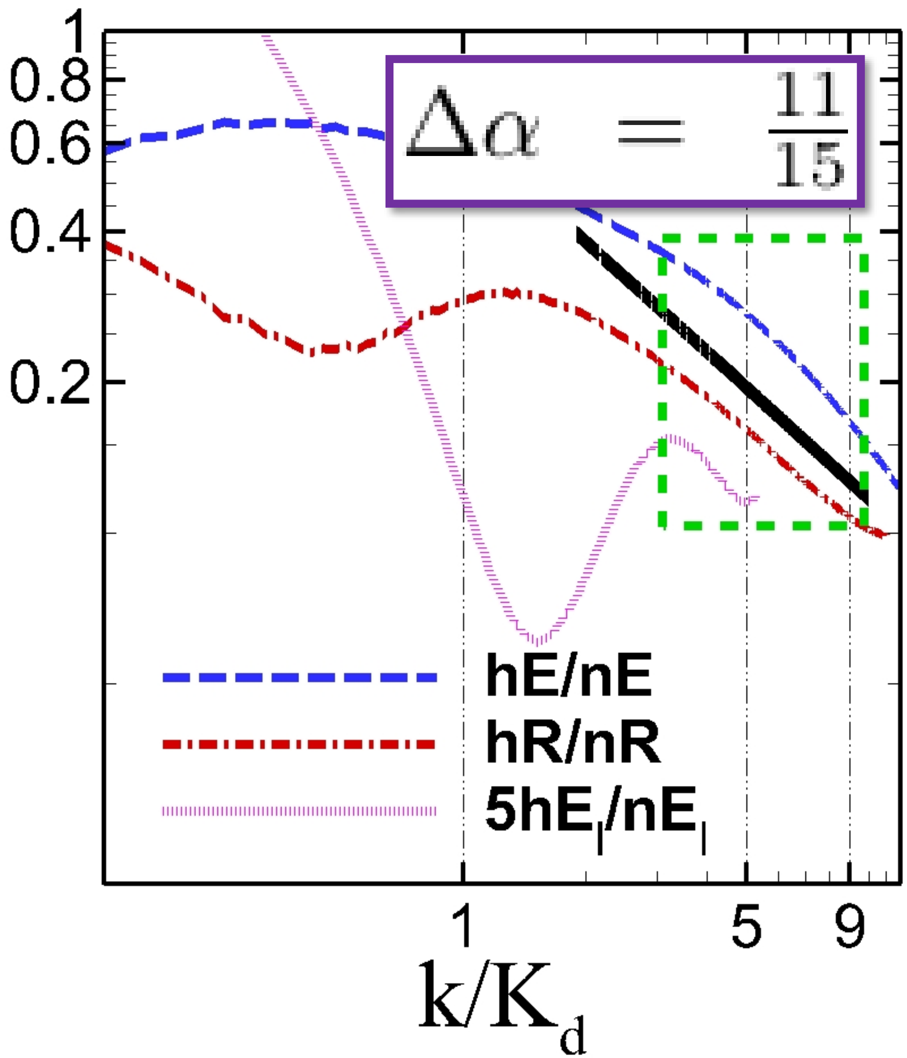


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Underlain by the “coherent” structure of the complex singularities of different variables: think about a shock

“Proof” (with numerical results of compressible turbulence driven by helical and nonhelical forces):

# 螺度约束扎紧湍流

with Yan YANG and Jun PENG @力学所
[acknowledgement: Jin-Xiu XU @无锡超算中心]

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**Ansatz**: I(k) is a power function (in the interested regime).

$$k_{dn} = 31.7 \qquad k_{dh} = 30 \qquad K_d = \sqrt{k_{dn}k_{dh}}$$

(the wavenumber denoting the beginning of the “visible” damping effect of viscosity, in units of lattice size 2\pi/1024 m for isotropic turbulence in a cyclic box)

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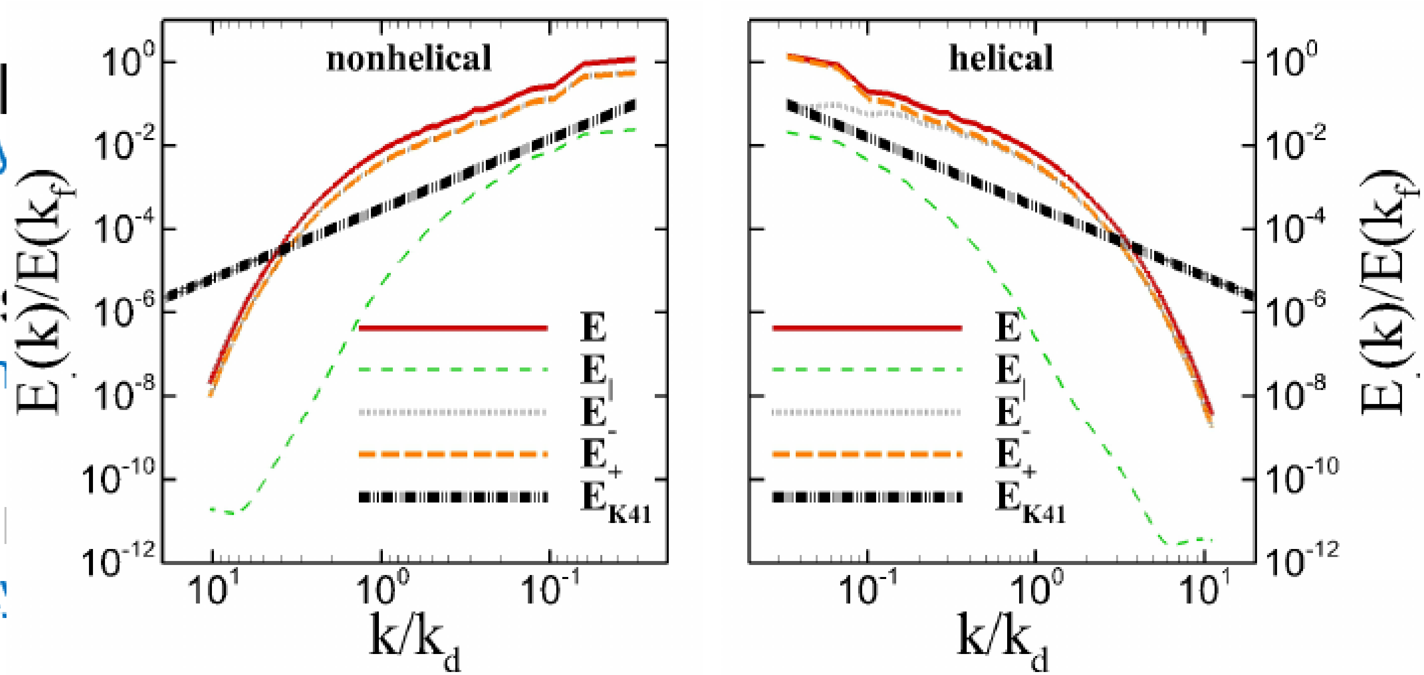


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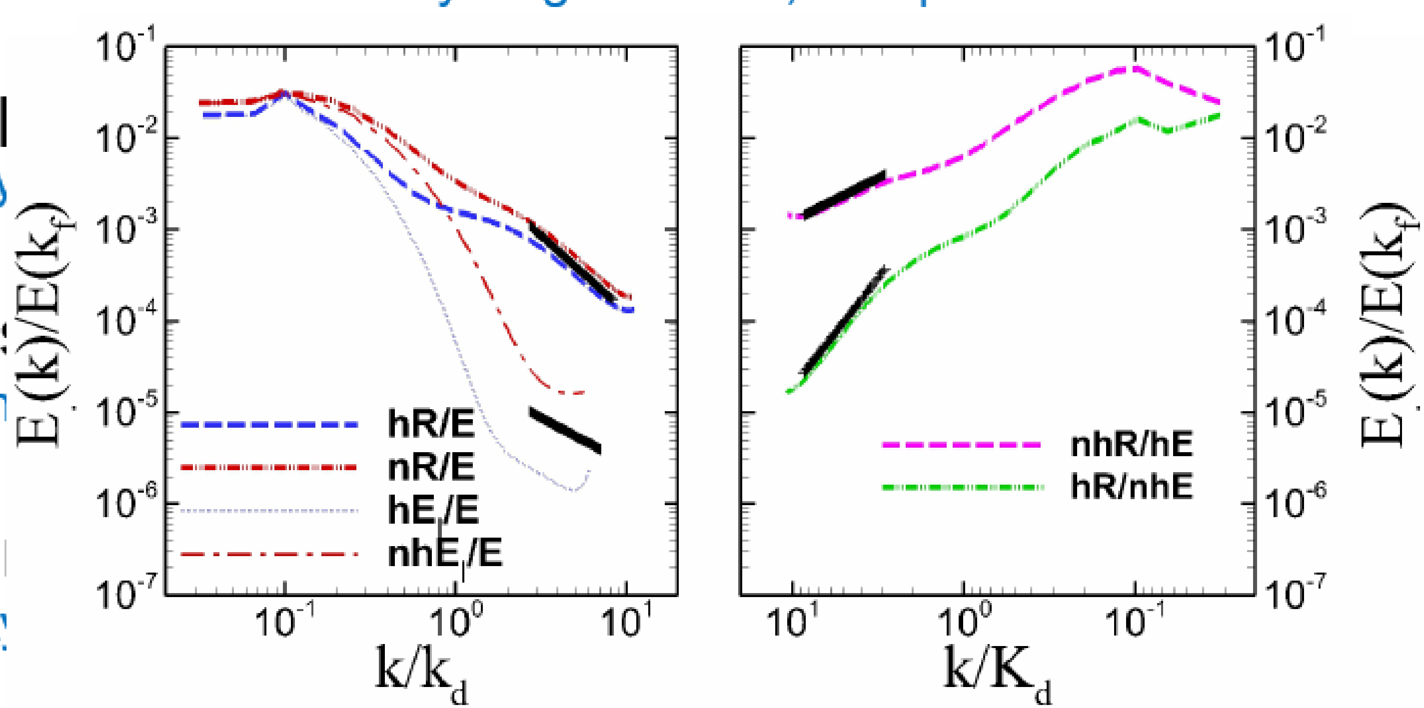


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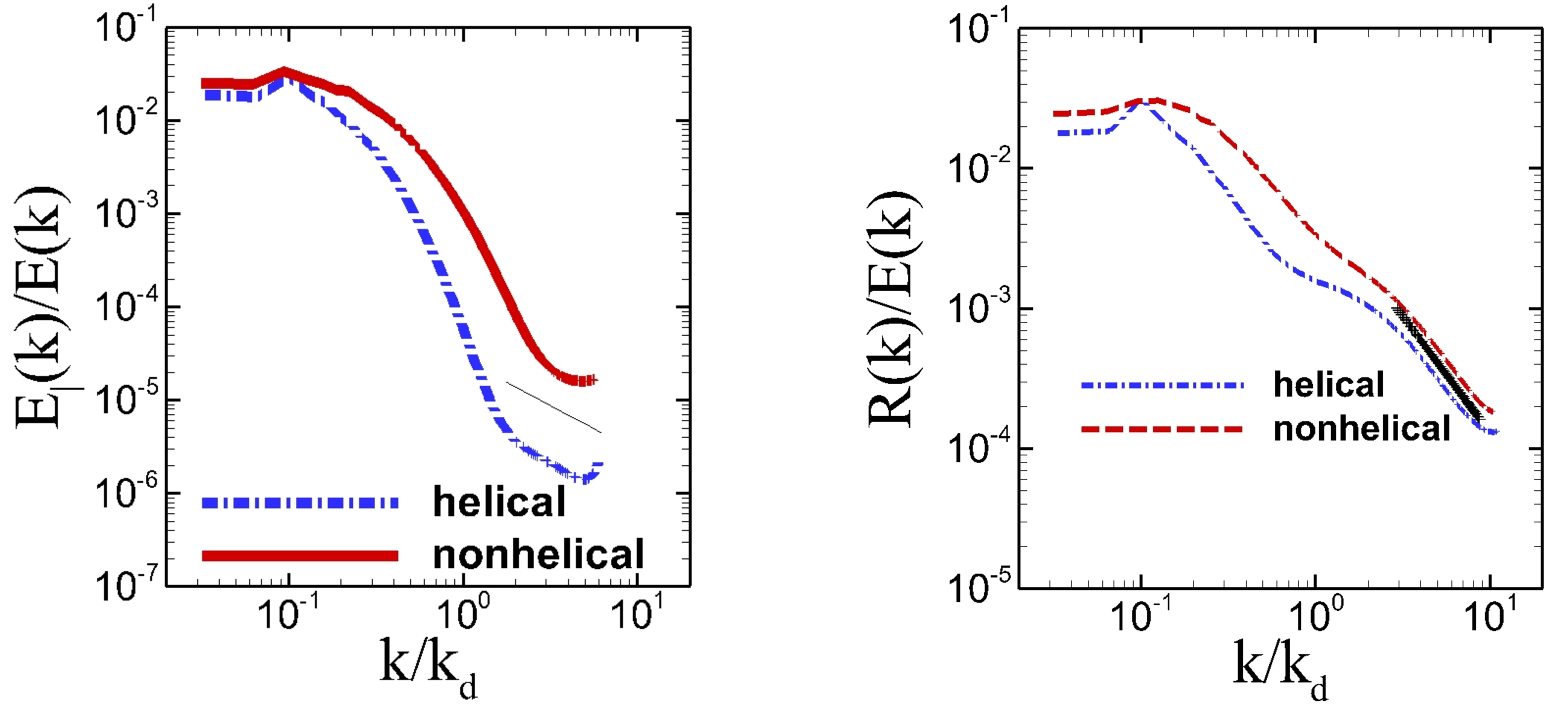


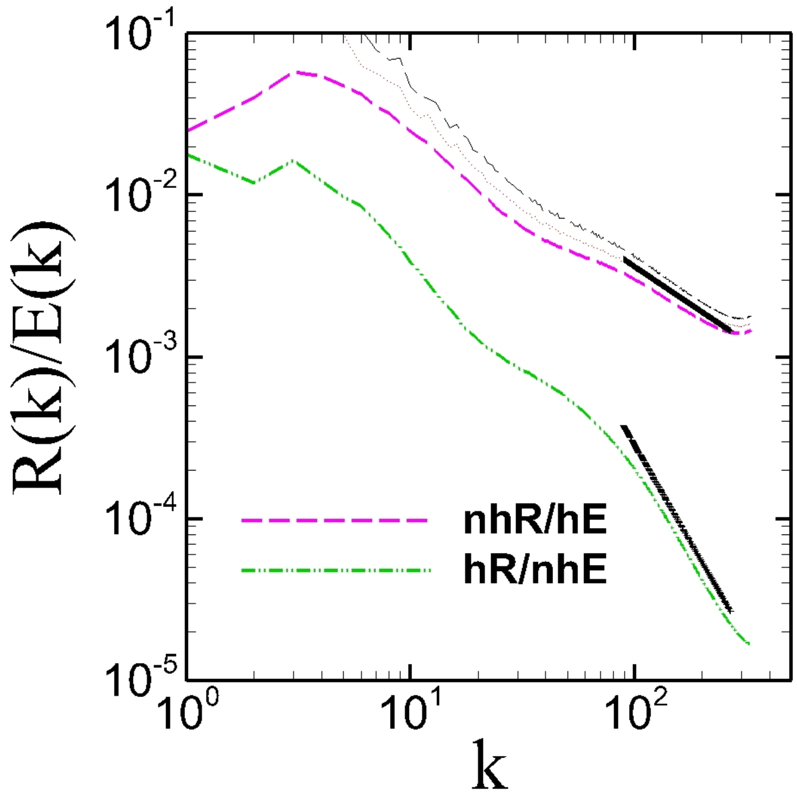

$$E_{|}(k)/E(k)\propto k^{-1} \text{ and } R(k)/E(k)\propto k^{-5/3}, \tag{9}$$

a new *universality* agreed by the helical and nonhelical cases. The above results indicate that, in either the helical or nonhelical case, the $E_{|}(k)$ and $R(k)$ are different from the $E(k)$ only up to power-law prefactors in the (stretch-)exponentially decaying in this dissipation range.

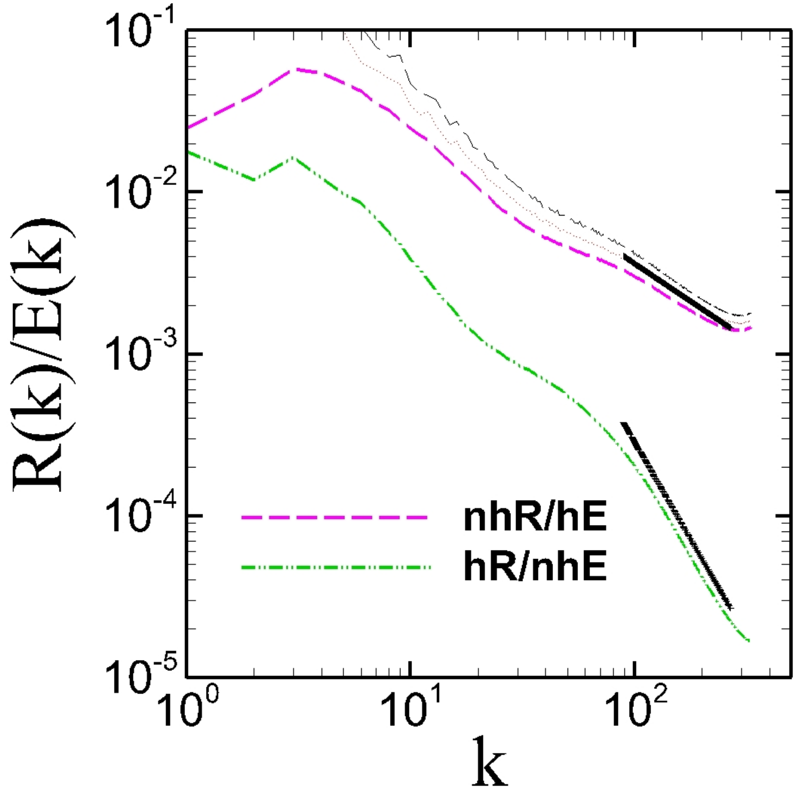
The observed Eq. (9) in this dissipation range indicates that, with again '$_h$(elical)' and '$_n$(onhelical)' and the self-evident superscripts,

$$F_h^R(k) = F_h^E(k),\ F_n^R(k) = F_n^E(k), \tag{11}$$

$$\alpha_h^R - \alpha_h^E = -5/3 = \alpha_n^R - \alpha_n^E. \tag{12}$$




$$R_h/E_n \propto k^{-12/5} \text{ and } R_n/E_h \propto k^{-14/15}. \tag{13}$$

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This observation means that,

$$F_h^R(k) = F_h^E(k),\ F_n^R(k) = F_n^E(k) \tag{14}$$

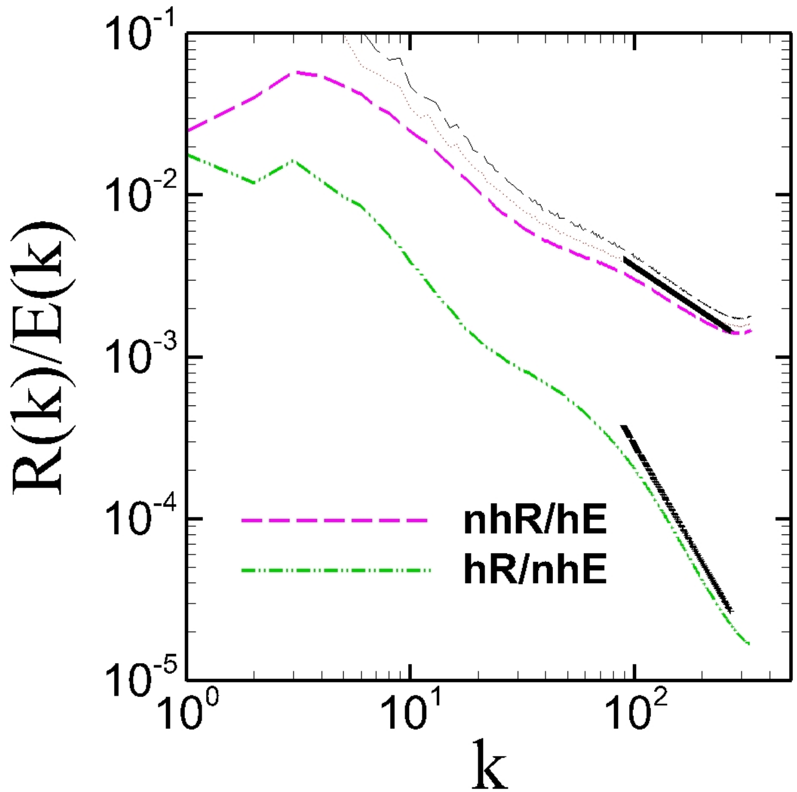
$$\text{and } \alpha_h^R - \alpha_n^E = -12/5,\ \alpha_n^R - \alpha_h^E = -14/15, \tag{15}$$

Thus, we deduce

$$F_h^R(k) = F_h^E(k) = F_n^R(k) = F_n^E(k) \tag{16}$$

$$\text{and } \Delta\alpha = \alpha_n^E - \alpha_h^E = 11/15 = \alpha_n^R - \alpha_h^R, \tag{17}$$

with all exponents determined up to a constant $\gamma$.

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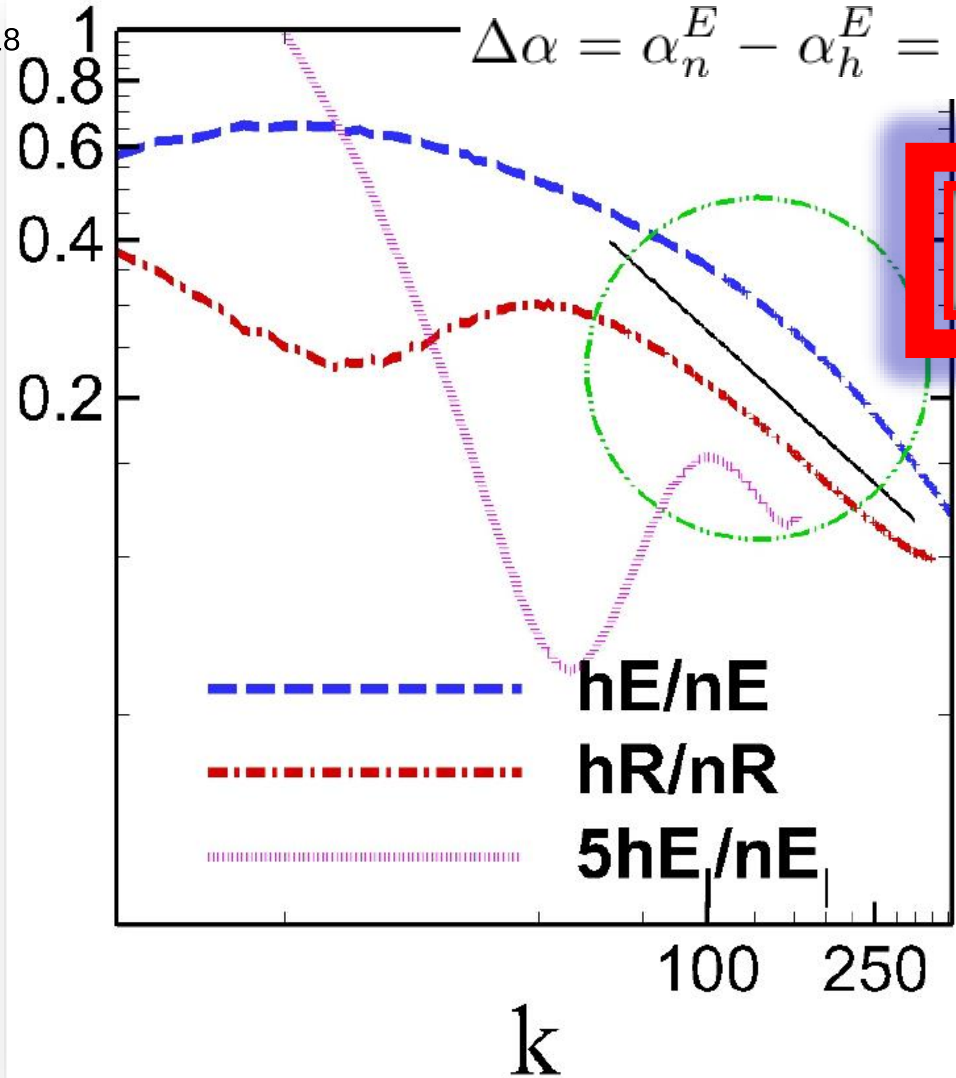
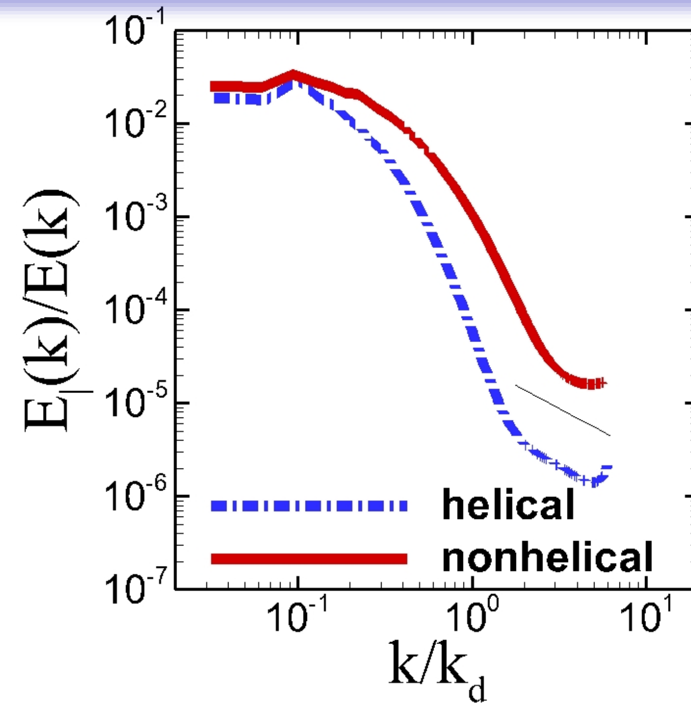
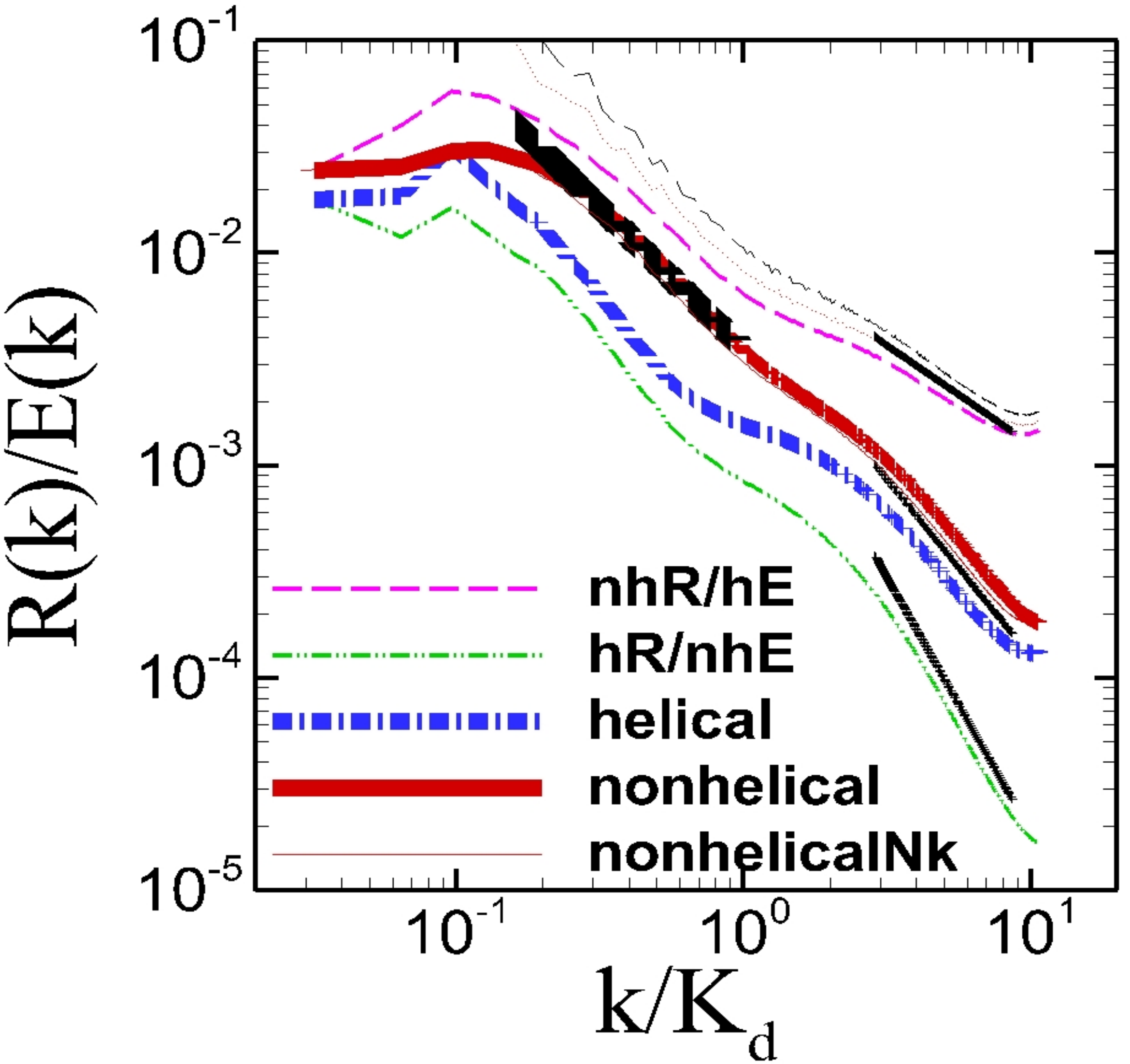


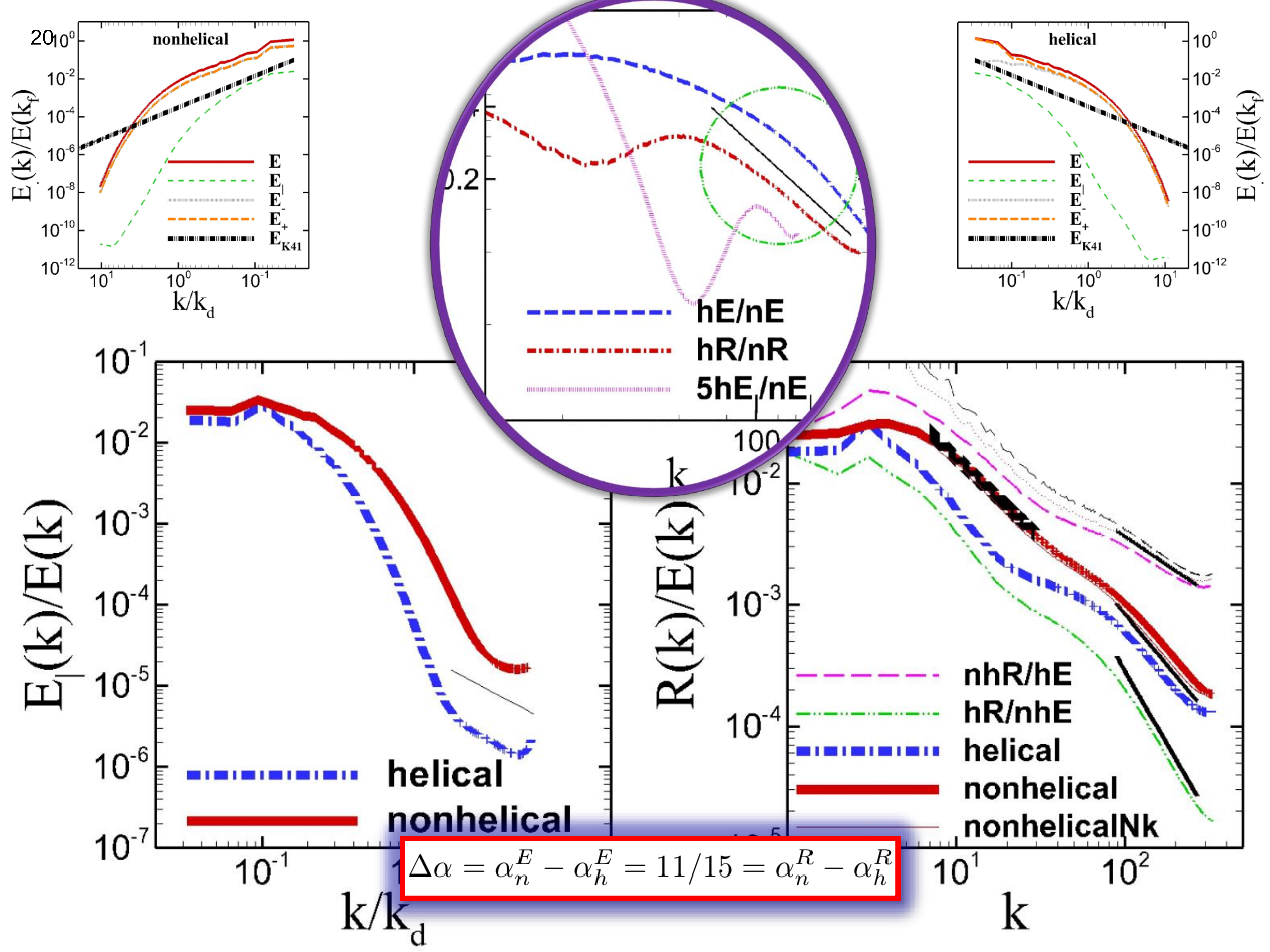

$$\boxed{\alpha_n^{E_|} - \alpha_h^{E_|} = \Delta\alpha = \frac{11}{15}}$$




The short straight line denotes the power law $\propto k^{-11/15}$ for $E_h(k)/E_n(k)$ ('hE/nE') and $R_h(k)/R_n(k)$ ('hR/nR'), verifying the result deduced and extending to $5E_{|h}(k)/E_{|n}(k)$ ('5hE$_|$/nE$_|$'), in the range circled out.

# Universal scaling in the inertial range?



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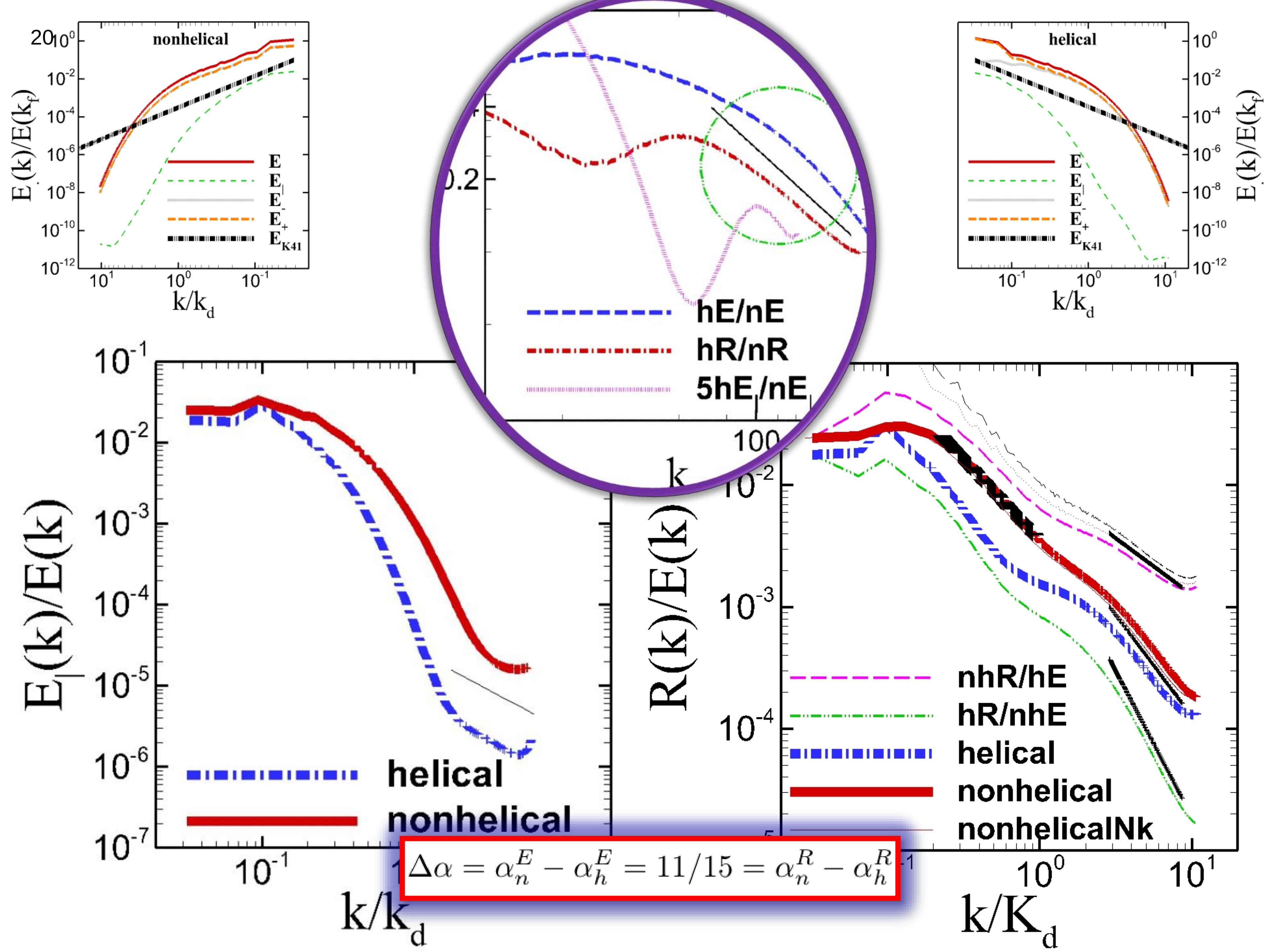

nonhelical
E
E_|
E_-
E_+
E_K41
E(k)/E(k_f)
k/k_d
helical
hE/nE
hR/nR
5hE_|/nE_|
0.2
100
k
E_|(k)/E(k)
helical
nonhelical
k/k_d
R(k)/E(k)
nhR/hE
hR/nhE
helical
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k/K_d
$\Delta\alpha = \alpha_n^E - \alpha_h^E = 11/15 = \alpha_n^R - \alpha_h^R$

Evaluation of the common part, $k^\gamma F(k)$, by fitting the spectra however can be subtle due to unknown *ansatz* for $F(k)$, and is not of our current interest.

Turbulence compressibility reduction with helicity is theoretically analyzed with the argument of complex singularities. Isotropic turbulence simulated in a cyclic box verifies smaller compressibility-relevant spectra in the case of helical, compared to nonhelical, large-scale forcing, with a difference of $\Delta\alpha = 11/15$ found in the power-law prefactor of the compressibility-relevant-mode spectrum $\propto k^\alpha F(k)$ in terms of the (normalized) wavenumber $k$: while $\alpha$ in the helical case is smaller, $F(k)$ is however the same as in the nonhelical one, indicating some weaker type of universality.

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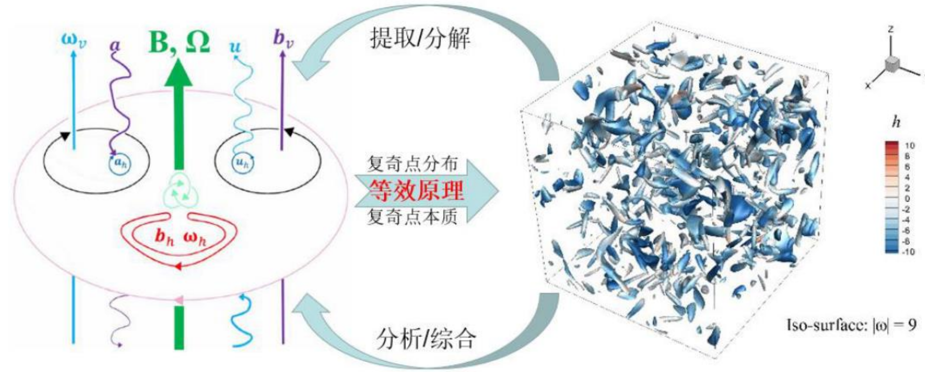
# (sub-)summary

- Less “compressive” in the helical case
- “Overall weaker complex singularities” (not a very precise expression, due to the random nature) measured by \Delta \alpha = 11/15 deep in the dissipation range
- Inertial range seems to be universal [with respective to the (helical) forcing scheme used] in the scaling exponent; universal/common *F* deep in the dissipation range




# 从手性基本流结构到有螺湍流

## Numerical *real Schur turbulence* in a cyclic box

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I stopped here due to time limit: some of the materials for this subsection are availabe from the references of the Abstract and from the link given at the bottom of the Outline page

# 致谢


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