Magnetic Helicity in Periodic Domains

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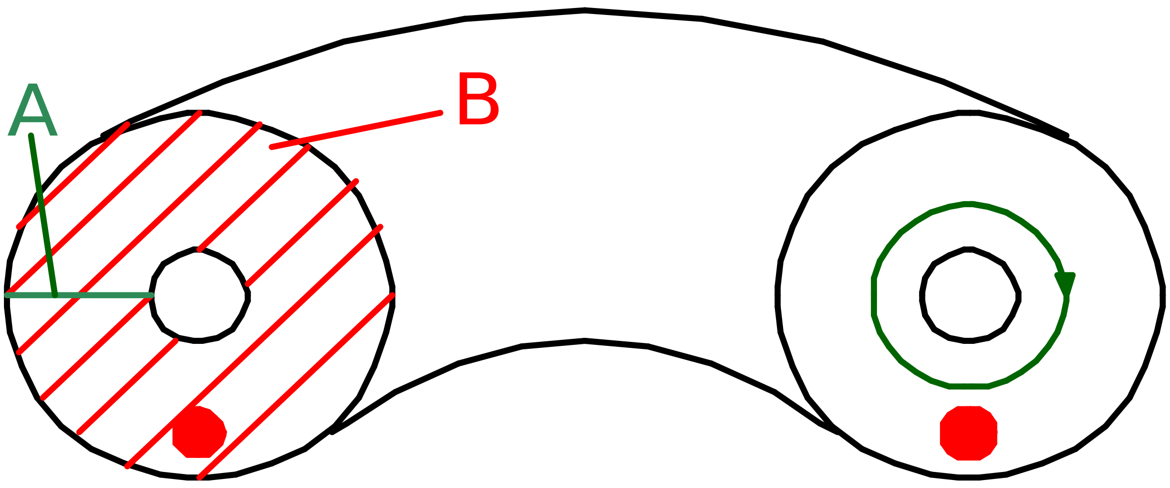
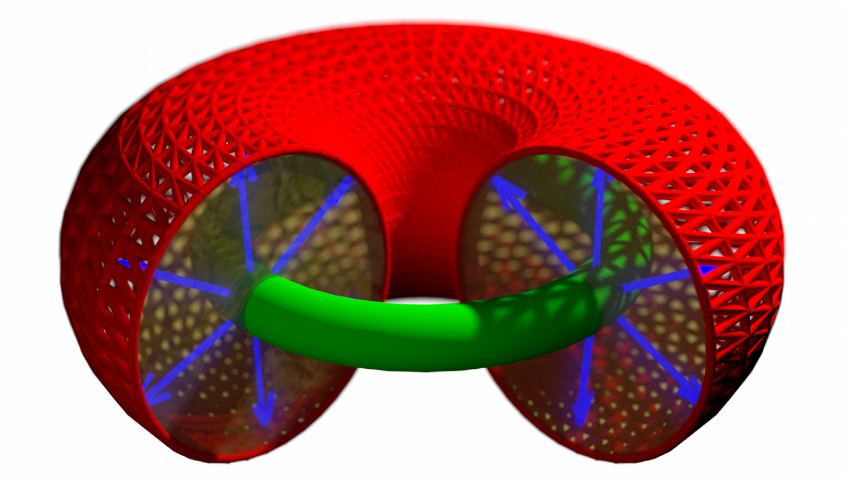


Existence of Vector Potentials

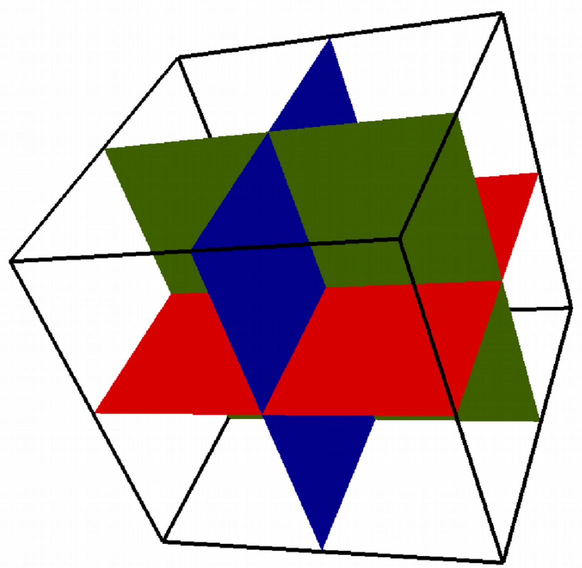
x-periodic: flux in xyz -> yes



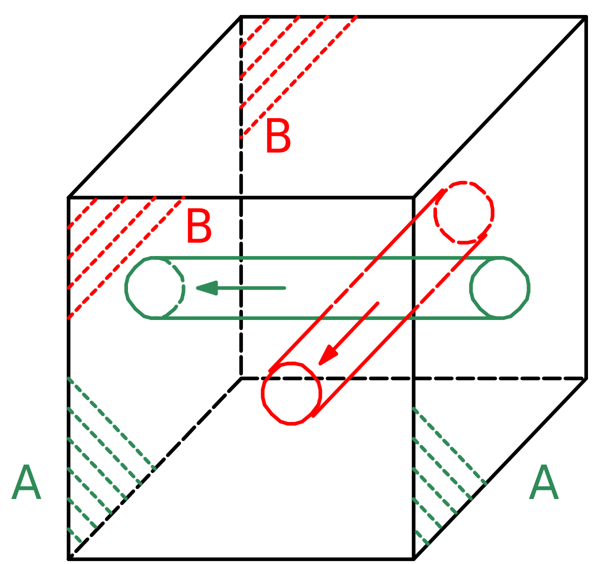
xy-periodic: flux in xy -> yes
flux in z -> no

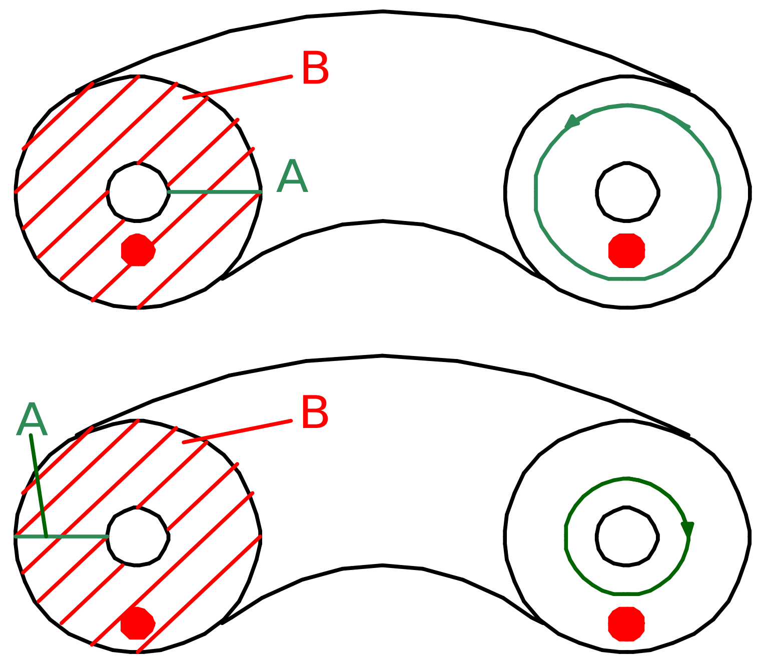
(Berger, 1996)

xyz-periodic: flux in xyz -> no



Doubly Periodic Domains

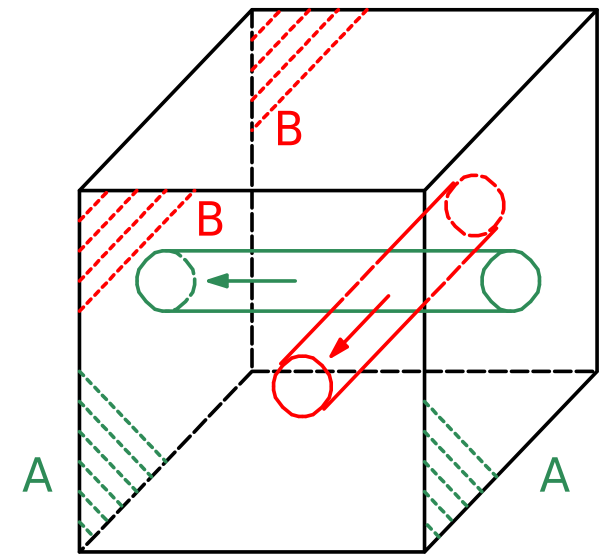




- Transformation into toroidal shell is ambiguous.
- Linking is destroyed.

Gauge Transform Helicity Away

- Localize magnetic field in tubes.
- Magnetic field is cylindrically symmetric.



→ Vector potential is poloidal around B:

$$\mathbf{A}_i \cdot \mathbf{B}_i = 0$$

$$H = \int \mathbf{A}_1 \cdot \mathbf{B}_2 + \mathbf{A}_2 \cdot \mathbf{B}_1 \, \mathrm{d}V = H_{12} + H_{21}$$

$$\mathbf{A}' = \mathbf{A} + \nabla\Psi_1$$

$$H'_{12} = H_{12} + \int \nabla\Psi_1 \cdot \mathbf{B}_2 \, \mathrm{d}V$$

Choose: $\nabla\Psi_1 \cdot \mathbf{B}_2 = \alpha_1 |\mathbf{B}_2|$

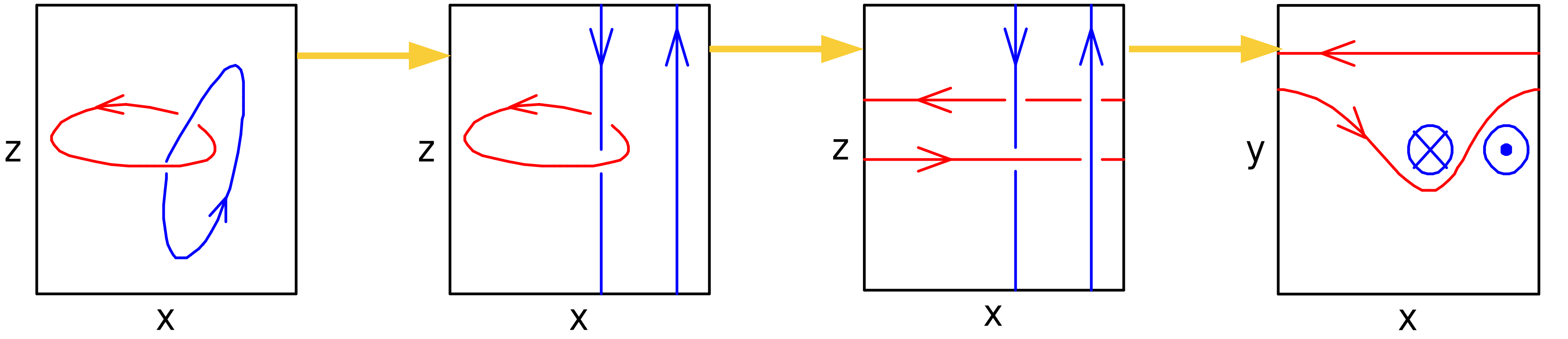
$$H'_{12} = H_{12} + \int \alpha_1 |\mathbf{B}_2| \, \mathrm{d}V$$

Choose: $\alpha_1 = -H_{12}/(\phi_2 L_z)$

$(\hat{\mathbf{B}}_2 = \mathbf{e}_z)$

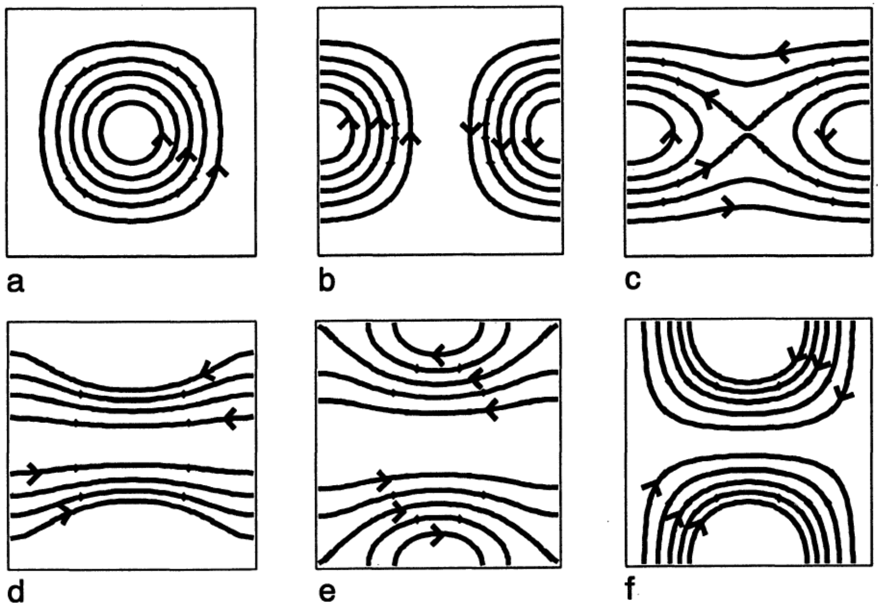
$$H'_{12} = 0$$

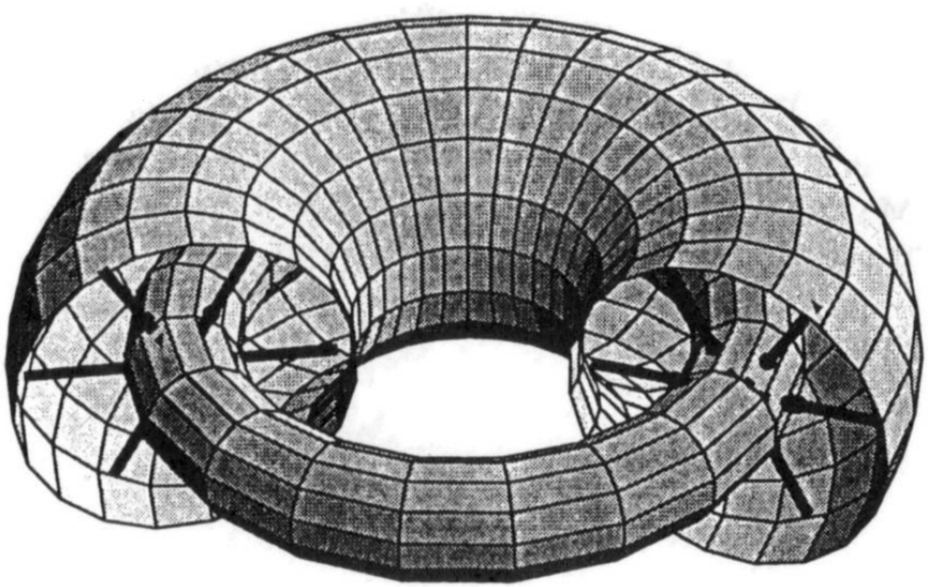
Zero Net-Flux in 3d Periodic



- Perform topology-conserving operations through the periodic boundaries.
- Last step: wind red flux tube around either vertical blue flux tube: same helicity.
- Zero net flux preserves helicity.

Flux in Doubly Periodic Domains





(Berger, 1996)

- Permissible operations lead to zero net helicity.
- However, vector potential does not exist.

Conclusions

- In periodic domains vector potentials do not always exist.
- Helicity is not always well defined.
- Lack of zero net-flux in periodic domain preserves helicity.

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