Magnetic helicity in rotating neutron stars consisting of electroweakly interacting inhomogeneous matter

MAXIM DVORNIKOV

IZMIRAN, RUSSIA

TOMSK STATE UNIVERSITY, RUSSIA

Plan of talk

Chiral phenomena

Berry phase

Electroweak interactions

Neutron stars properties

Effective action for chiral electrons in electroweak matter

Corrections to the anomalous current and Adler anomaly

Helicity flow in stars

Applications for magnetic cycles of pulsars

Conclusion

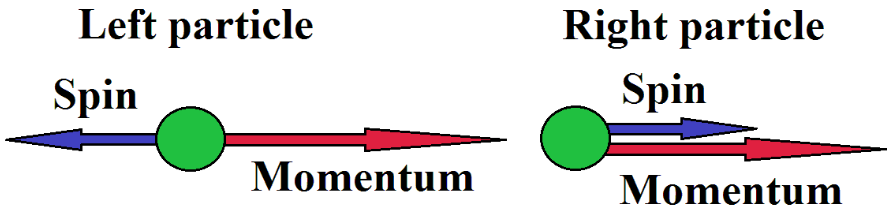
Publications

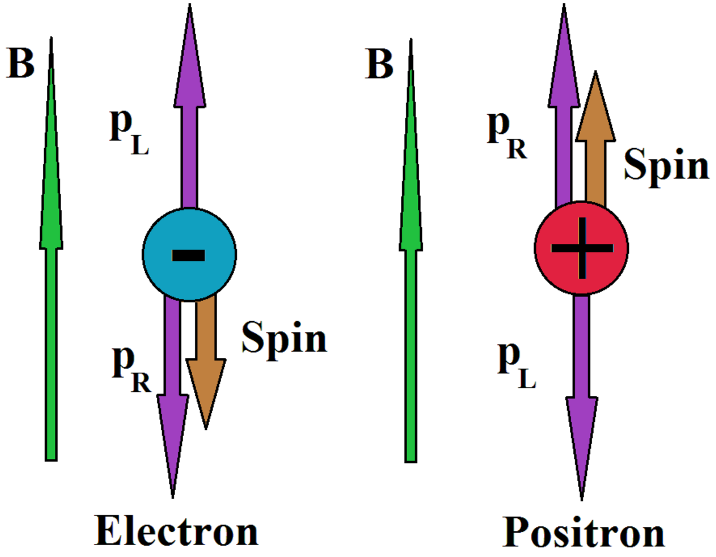
M. Dvornikov, *Magnetic helicity in plasma of chiral fermions electroweakly interacting with inhomogeneous matter*, Nucl. Phys. B 955, 115049 (2020), arXiv:2005.09358.

M. Dvornikov, *Galvano-rotational effect induced by electroweak interactions in pulsars*, JCAP 05 (2015) 037, arXiv:1503.00608.

Chiral magnetic effect (CME)

Helicity is strongly correlated with the momentum for massless particles





While interacting with a constant magnetic field **B**, the spin of a charged particle (e.g. an electron) is aligned opposite **B** and the spin of an antiparticle (a positron) along B, at zero Landau level.

Left electrons move along **B**, whereas right ones opposite **B**.

Thus we can expect a flux of charged particles, i.e. electric current, along **B**.

Vilenkin (1980) showed

$$\mathbf{J} = \frac{2\alpha_{em}}{\pi}\mu_5\mathbf{B}, \quad \mu_5 = \frac{1}{2}(\mu_R - \mu_L)$$

Chiral vortical effect

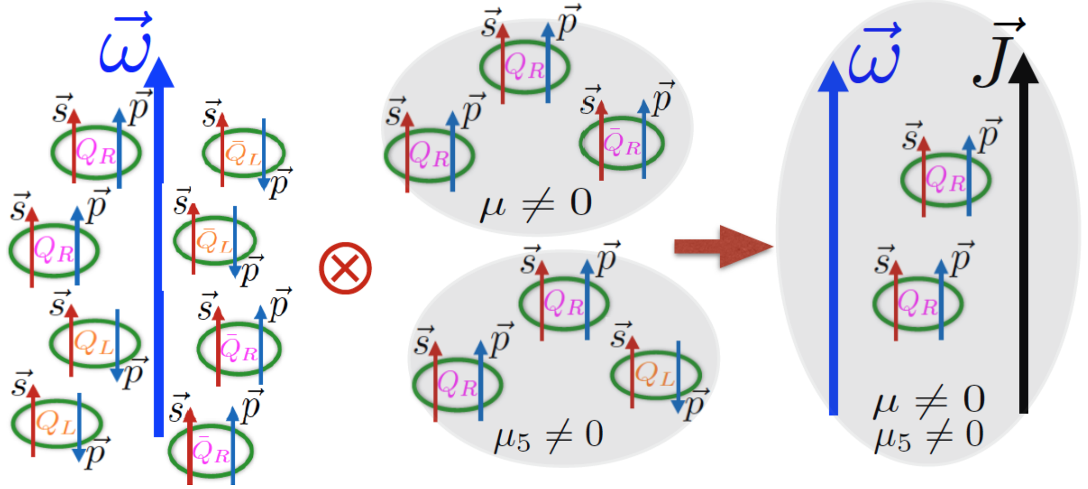
In presence of rotation, fermions experience the interaction $-(\vec{s}\vec{\omega})$

Thus, there is a spin polarization $\langle\vec{s}\rangle \propto \vec{\omega}$, which is charge-blind

If $\mu_5 \neq 0$, there are unequal number right and left particles, which move along the rotation axis since $\langle\vec{p}\rangle \propto \langle\vec{s}\rangle \propto \vec{\omega}$

If $\mu \neq 0$, there are unequal number of particles and antiparticles

If both $\mu_5 \neq 0$ and $\mu \neq 0$, there is a current along the rotation axis $\vec{J} \propto \mu\mu_5\vec{\omega}$

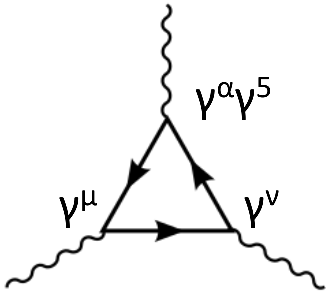


Adler anomaly

The currents of left and right particles $j^{\mu}_{R,L} = \left(n_{R,L}, \vec{j}_{R,L}\right)$ are conserved in classical physics:

$$\partial_\mu j^{\mu}_{R,L} = \partial_t n_{R,L} + \nabla \vec{j}_{R,L} = 0$$

In the presence of external electromagnetic field and accounting for the quantum corrections, this conservation law is violated



$$\partial_\mu \left(j^{\mu}_R - j^{\mu}_L \right) = \frac{2\alpha_{em}}{\pi} \left(\mathbf{E} \cdot \mathbf{B} \right)$$

Evolution of the magnetic helicity results from Maxwell equations

$$\frac{dh(t)}{dt} = -\frac{2}{V} \int d^3x \left(\mathbf{E} \cdot \mathbf{B} \right)$$

Integrating this eq. over isotropic space, we get the conservation law

$$\frac{d}{dt}\left[n_R - n_L + \frac{\alpha_{em}}{\pi} h(t) \right] = 0$$

Berry phase

Schrodinger equation with the time dependent Hamiltonian cannot be solved in general case

$$i\frac{d\psi}{dt} = \widehat{H}(t)\psi$$

If we suppose a slow change of H(t) through the parameters λ_i(t), the approximate solution has the form (Berry, 1983),

$$\psi(t) \approx \exp[-i\Theta(t)]\psi(0)$$

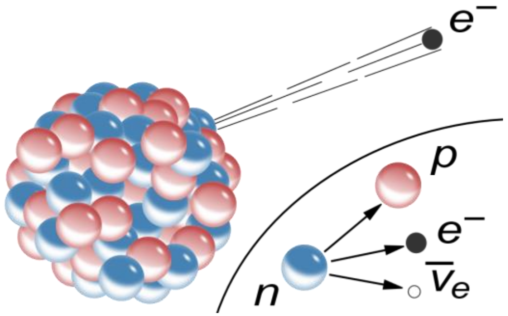
The Berry phase reads

$$\Theta(t) = \int \mathcal{A}_i d\lambda_i\,, \qquad \mathcal{A}_i = -iu^\dagger \frac{\partial u}{\partial \lambda_i}\,, \qquad \widehat{H}u = Eu$$

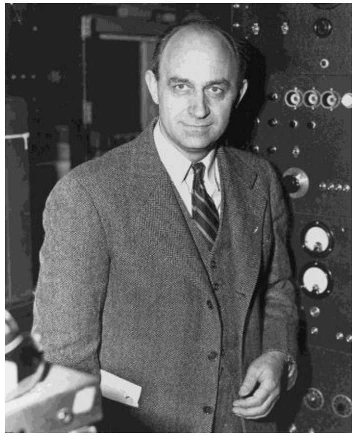
$\vec{\mathcal{A}}$ is the Berry curvature

Weak interaction

Beta-decay



Fermi theory: 1933

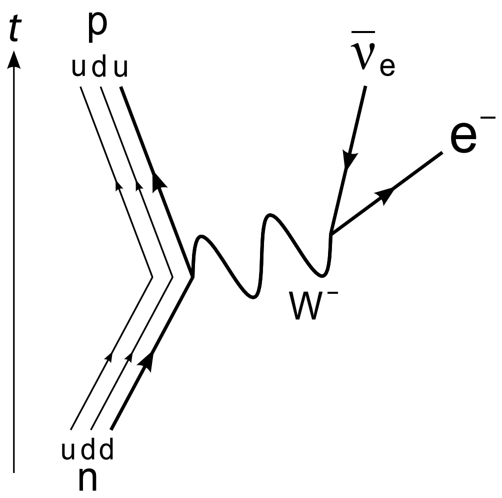


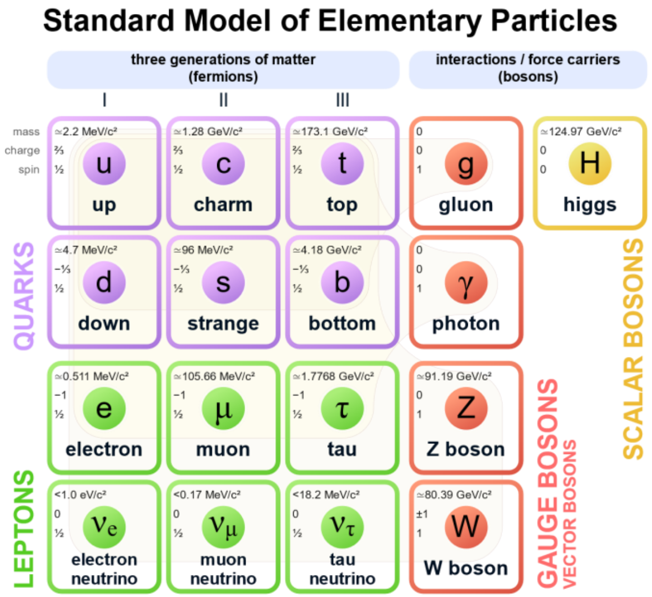
Hamiltonian = (Fermi constant) x (Current) x (Current)

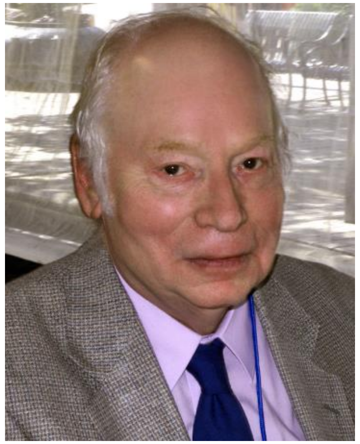
Parity is violated. Confirmed in Wu experiment: 1956

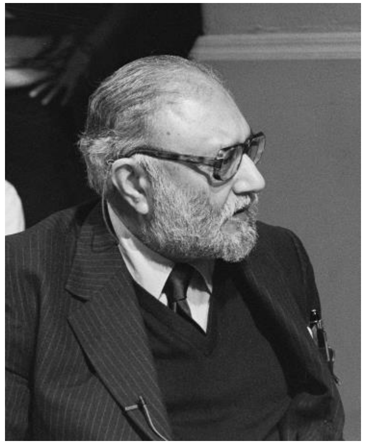
Electroweak interaction

- Weinberg-Salam theory: 1964
- Discovery of W and Z bosons: 1983



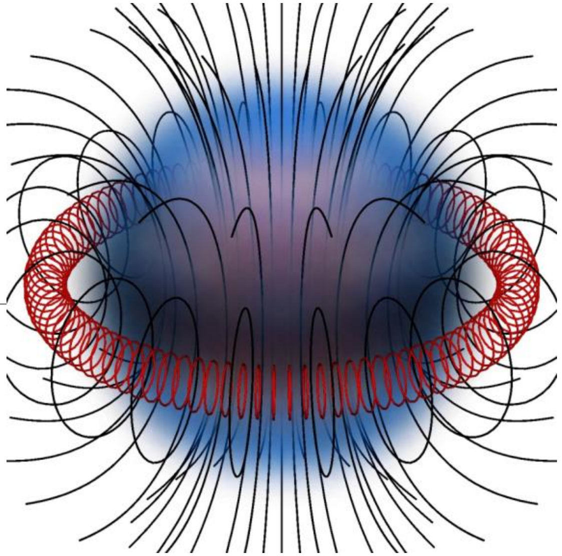






- Discovery of the Higgs boson: 2012
- Fermi approximation is sufficient for astrophysical applications

Properties of NSs



Size: 10 km

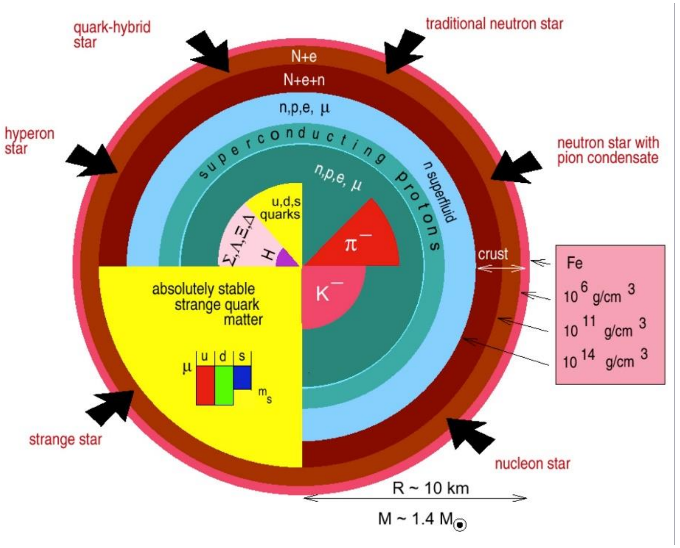
Mass: (1.4 – 2) M_{SUN}

Density: 10^{14} g/cm^3

Magnetic field: 10^{12} G

Consists mainly of neutrons with up to several % fraction of electrons and protons

Pulsars emit EM radiation with period (1 – 10^{-3}) s



Applicability of chiral phenomena for electrons in neutron stars

Fermi momentum of electrons in NS matter $p_F \sim 100\ MeV$. It corresponds to the density $n_e \sim 10^{36} cm^{-3}$.

Such electrons are ultrarelativistic since $p_F \gg m_e$.

We can neglect electron mass and assume that electrons are chiral particles.

Left and right electrons evolve independently

Electroweak interaction of chiral electrons with NS matter

NS consists mainly of neutrons with $n_n \sim 10^{38} cm^{-3}$

Right and left chiral electrons interact with neutrons with the effective potentials $V_{R,L}^{\mu}$, where $V_R^{\mu} \neq V_L^{\mu}$

$V_{R,L}^0 \sim G_F n_n$ and $\vec{V}_{R,L} \sim V_{R,L}^0 \vec{v}$

If NS rotates, $\vec{v} = (\vec{\omega} \times \vec{r})$

Effective action of chiral electrons

Effective energy of a chiral electron in matter

$$E_{eff} = E_{R,L} + \Theta_{R,L}, \qquad E_{R,L} = V_{R,L} + |\vec{p} - \vec{V}_{R,L}|$$

Effective action for electrons in matter and electromagnetic field

$$S_{R,L} = \int_{t_0}^{t} dt \left\{ \dot{\vec{x}}\left[\vec{P} + e\vec{A}_{eff}\right] - \vec{\mathcal{A}}_{R,L}\dot{\vec{P}} - \epsilon_{R,L} - eA_0 \right\}$$

$$\vec{A}_{eff} = \vec{A} + \vec{V}_{R,L}/e$$

$\vec{\mathcal{A}}_{R,L}$ is the Berry curvature

Noncommutative dynamics

If Berry curvature is present, the Poisson brackets are (Son & Yamamoto, 2013)

$$\{P_i, P_j\} = -\frac{\varepsilon_{ijk} B_k^{(eff)}}{1 + \vec{B}_{eff}\vec{\Omega}}, \qquad \{x_i, x_j\} = \frac{\varepsilon_{ijk}\Omega_k}{1 + \vec{B}_{eff}\vec{\Omega}} \neq 0$$

$$\{P_i, x_j\} = \frac{\delta_{ij} + \Omega_i B_j^{(eff)}}{1 + \vec{B}_{eff}\vec{\Omega}}$$

$$\vec{\Omega} = \nabla_P \times \vec{\mathcal{A}}_{R,L} = \pm\frac{\vec{P}}{2P^2}, \qquad \vec{B}_{eff} = \nabla \times \vec{A}_{eff}$$

Correction to the electric current

Basing on the effective action, we derive the kinetic equation for right and left electrons

Using the distribution functions and assuming that electrons are degenerate, we get the currents in the form

$$\vec{J}_{R,L} = \pm \frac{e^2}{4\pi^2} \mu_{R,L} \left[\vec{B} + \frac{1}{e} \nabla \times \vec{V}_{R,L} \right]$$

The first term reproduces the CME

The second term is the correction owing to the electroweak interaction with (rotating) matter (see also Dvornikov (2015))

Correction to the Adler anomaly

Analogously to the electric current correction, one get the correction to the Adler anomaly

$$\frac{dn_5}{dt} = \frac{e^2}{2\pi^2 V}\int \left[\left(\vec{E}\vec{B}\right) - \frac{1}{e}(\vec{E}\cdot\nabla\times\vec{V}_5)\right] d^3x - \Gamma n_5$$

$$n_5 = n_R - n_L, \qquad \vec{V}_5 = \vec{V}_L - \vec{V}_R, \qquad \nabla\times\vec{V}_5 \sim G_F n_n \vec{\omega}$$

Analogous correction in case of the neutrino gas was obtained by Dvornikov & Semikoz (2017,2018)

We add the chiral imbalance relaxation term, $-\Gamma n_5$, which describes the transitions between left and right electrons in their collisions because of small but nonzero mass.

Electroweak contribution to the helicity evolution

Chiral imbalance vanishes very rapidly in NS: $n_5 \to 0$

Helicity evolution

$$\frac{dH(t)}{dt} = -2\int d^3x(\mathbf{E}\cdot\mathbf{B})$$

Electroweak contribution to the helicity change

$$\left(\frac{dH(t)}{dt}\right)_{EW} = -2\frac{V_5}{e}\int d^3x\left(\vec{E}\cdot\vec{\omega}\right)$$

We use the Maxwell equation $\vec{E} = \frac{\vec{J}}{\sigma} = {}^{\nabla\times\vec{B}}/_{\sigma}$

$$= -2\frac{V_5}{e\sigma}\int d^3x\left(\nabla\times\vec{B}\cdot\vec{\omega}\right)$$

$$V_5 \sim G_F n_n$$

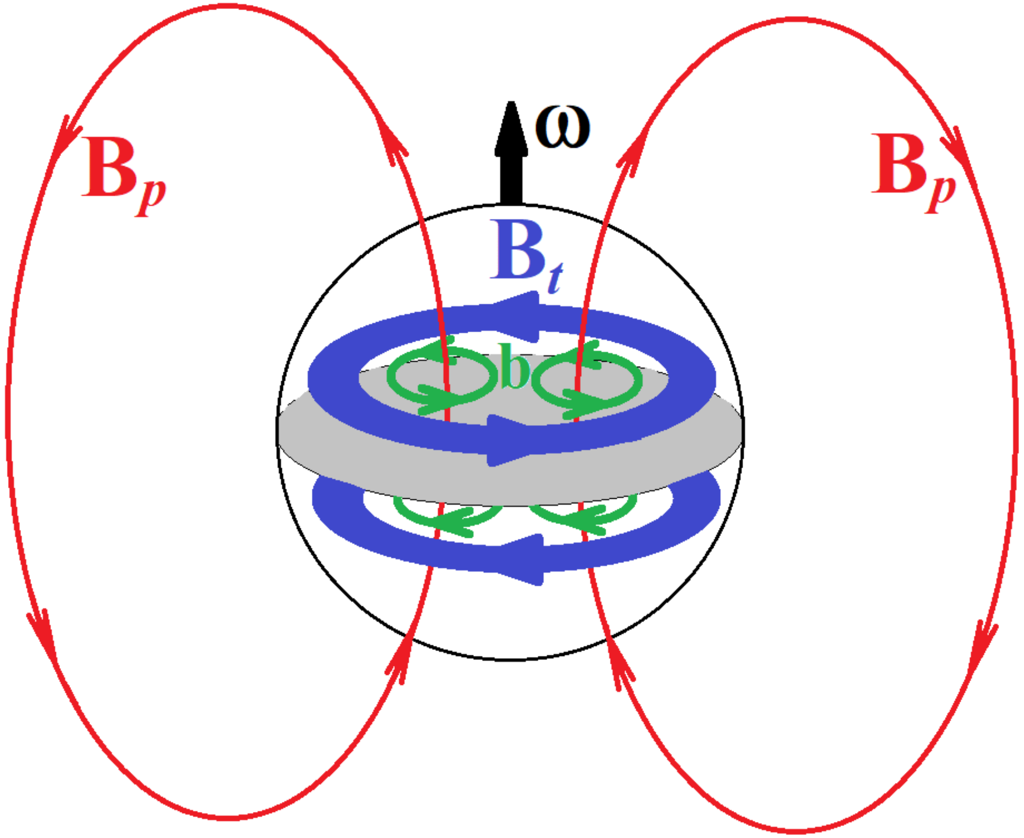
Application for NS

We require that $\nabla \times \vec{B} \parallel \vec{\omega}$

It happens when toroidal component of **B** is present

Total helicity is conserved: $\dot{H} = \dot{H}_N + \dot{H}_S = 0$

There is a helicity flow through the equator



Implementation in NS

We suppose that *N* vortices of small scale magnetic field **b** detach from the toroidal field because of the magnetic fields turbulence

Assuming the flux conservation, we get $br^2N = B_t R_t^2$

Estimate for

$$\left(\nabla \times \vec{B}\right) \sim \frac{B_t R_t^2}{r^3}$$

Typical time for the helicity change

$$\tau \sim \frac{H_N}{|\dot{H}_N|} = \frac{3}{4\pi} \frac{eB_p \sigma r^3}{\omega V_5 R}$$

Relation to the magnetic cycles of pulsars

Brandenburg et al. (2013) found that the helicity flow though the equator of the Sun is related to the 22 yr solar cycle

Gusakov et al (2016) obtained that magnetic flux tubes with $r = 6 \times 10^{-5}$ cm can appear in NS matter

We take $B_p = 10^{12}$ G, $\sigma = 10^6$ Gev (Schmitt et al., 2018), $V_5 = 6$ eV, $\omega = 10^3$ s^{-1}, and $R = 10$ km

We get that $\tau = 4 \times 10^3$ yr

Such typical time is close to periods of magnetic cycles of some pulsars $\tau = (10^3 - 10^4)$ yr observed by Contopoulos (2007)

Conclusion

We found the Berry phase and the effective actions for chiral electrons electroweakly interacting with inhomogeneous (rotating) matter

The contributions to the anomalous current and to the Adler anomaly were obtained

The electroweak correction leads to the helicity flow though the equator of a star

The typical time is close to periods of magnetic cycles of some pulsars if we take realistic parameters of NSs

Acknowledgments

Russian Science Foundation
(Grant No.19-12-00042)