Magnetic winding – a key to unlocking topological complexity in flux emergence

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Joint work with **Chris Prior** (Durham)

Overview

Introduction

Winding

Flux emergence

Summary & Discussion

Helicity Library

Solar magnetic fields

Figure: Twisted solar prominence (SDO)

Field line topology

- Twisted field $\rightarrow$ coronal mass ejections (CMEs)
- Braided field $\rightarrow$ reconnection & flares

In order to understand how eruptions form, we need to understand their field line topology. This, of course, is where **helicity** enters.

Classical helicity

In a simply connected domain Ω, with $\boldsymbol{B} \cdot \boldsymbol{n} = 0$ on $\partial\Omega$, the *magnetic helicity* is defined as

$$H = \int_{\Omega} \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V,$$

where $\nabla \times \boldsymbol{A} = \boldsymbol{B}$.
H is an invariant of ideal MHD and, with the above boundary conditions, is gauge invariant, i.e. invariant to the transformation $\boldsymbol{A} \rightarrow \boldsymbol{A} + \nabla\phi$.

Coulomb gauge

In order to link *helicity* to *topology*, we need to consider a particular gauge. Using the **Coulomb gauge**

$$\nabla \cdot \boldsymbol{A} = 0,$$

the helicity can be written in the form

$$H = \frac{1}{4\pi} \int_\Omega \int_\Omega \boldsymbol{B}(\boldsymbol{x}) \cdot \boldsymbol{B}(\boldsymbol{y}) \times \frac{\boldsymbol{x} - \boldsymbol{y}}{|\boldsymbol{x} - \boldsymbol{y}|^3} \, \mathrm{d}^3 x \, \mathrm{d}^3 y.$$

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Where is the topology here?

Hopf link

Consider the magnetic field confined to two "thin" linked loops.

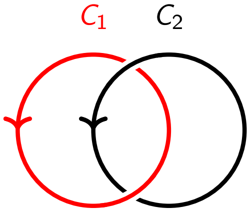

Figure: Two loops C_1 and C_2 linked once with given directions.

Let the fluxes be Φ_1 and Φ_2.

Gauss linking number

The key quantity here is the *Gauss linking number*

$$Lk(\mathbf{x},\mathbf{y}) = \frac{1}{4\pi}\int_{\mathbf{x}(s)}\int_{\mathbf{y}(\sigma)} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}s}\cdot\frac{\mathrm{d}\mathbf{y}}{\mathrm{d}\sigma}\times\frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3}\,\mathrm{d}s\,\mathrm{d}\sigma.$$

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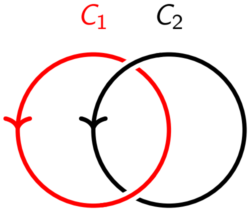
Helicity is *Gauss linkage* **weighted by** *magnetic flux*.

Back to the Hopf link



The Gauss linking number is $Lk_{12} = Lk_{21} = -1$. With fluxes Φ_1 and Φ_2, $H = 2Lk_{12}\Phi_1\Phi_2 = -2\Phi_1\Phi_2$.

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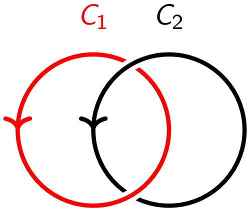

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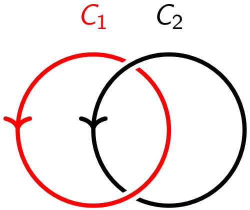

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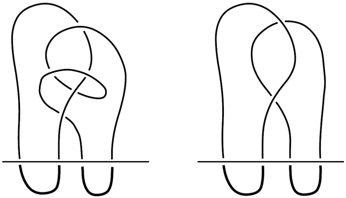
However, the fields for solar eruptions of interest are not closed...

Relative helicity

- Magnetic fields in the solar atmosphere are connected to the lower boundary ($\boldsymbol{B} \cdot \boldsymbol{n} \neq 0$).
- We need a different meaure of helicity.
- We use *relative helicity* H_R, which compares the "entanglement" of two different magnetic fields with the same boundary conditions.



Arnold & Khesin 1998

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Let $\boldsymbol{B}$ be the main field and $\boldsymbol{B}'$ be the reference field with $\boldsymbol{B} \cdot \boldsymbol{n} = \boldsymbol{B}' \cdot \boldsymbol{n}$ on $\partial\Omega$.

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$$H_R = \int_{\Omega} (\boldsymbol{A} + \boldsymbol{A}') \cdot (\boldsymbol{B} - \boldsymbol{B}') \, \mathrm{d}V,$$

where $\nabla \times \boldsymbol{A} = \boldsymbol{B}$ and $\nabla \times \boldsymbol{A}' = \boldsymbol{B}'$.

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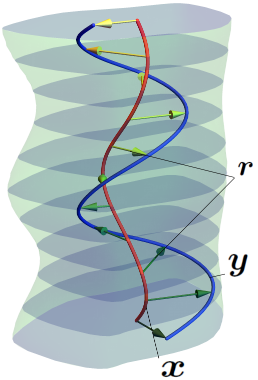
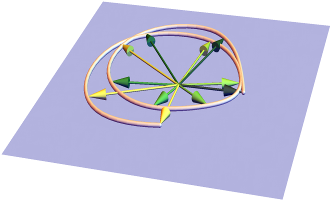
In order to understand the underlying topological structure of H_R, we need to consider a specific gauge (just as for classical helicity).

Winding - the basics

Consider a Euclidean space with the standard Cartesian basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_z\}$. Let P be a horizontal plane and consider two curves, $\boldsymbol{x}$ and $\boldsymbol{y}$, monotonically increasing in the $\boldsymbol{e}_z$-direction.





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The winding has the expression

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{2\pi} \int_0^h \frac{\mathrm{d}}{\mathrm{d}z} \theta(\boldsymbol{x}(z), \boldsymbol{y}(z)) \, \mathrm{d}z.$$

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This can be integrated to give

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2\pi}[\theta(\boldsymbol{x}(h),\boldsymbol{y}(h)) - \theta(\boldsymbol{x}(0),\boldsymbol{y}(0))] + n,$$

where $n \in \mathbb{Z}$ is the number of full rotations of the "joining vector" (from the diagram) around the origin.

Non-monotonic curves

What about curves that are not monotonically increasing in height?

Non-monotonic curves

What about curves that are not monotonically increasing in height? Suppose $\boldsymbol{x}$ and $\boldsymbol{y}$ have n and m distinct turning points respectively, that is points where $\mathrm{d}x_z/\mathrm{d}z = 0$ ($\boldsymbol{x} \cdot \boldsymbol{e}_z = x_z$) or $\mathrm{d}y_z/\mathrm{d}z = 0$ ($\boldsymbol{y} \cdot \boldsymbol{e}_z = y_z$). Now split $\boldsymbol{x}$ into $n+1$ regions and $\boldsymbol{y}$ into $m+1$ regions. In each region, curve sections $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$ share a mutual z-range $[z_{ij}^{\min}, z_{ij}^{\max}]$. Hence, in each section, monotonic winding can be applied and the total winding can be written as

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \frac{\sigma(\boldsymbol{x}_i)\sigma(\boldsymbol{x}_j)}{2\pi} \int_{z_{ij}^{\min}}^{z_{ij}^{\max}} \frac{\mathrm{d}}{\mathrm{d}z} \theta(\boldsymbol{x}_i(z), \boldsymbol{y}_j(z))\, \mathrm{d}z,$$

where $\sigma(\boldsymbol{x}_i)$ is an indicator function marking where the curve section $\boldsymbol{x}_i$ moves up or down in z, i.e.

$$\sigma(\boldsymbol{x}_i) = \begin{cases} 1 & \text{if} \quad \mathrm{d}x_z/\mathrm{d}z > 0, \\ -1 & \text{if} \quad \mathrm{d}x_z/\mathrm{d}z < 0, \\ 0 & \text{if} \quad \mathrm{d}x_z/\mathrm{d}z = 0. \end{cases}$$

Winding helicity

In our "stacked" domain, we can find a useful vector potential. For reasons soon to be discussed,

$$\boldsymbol{A}^W(x_1, x_2, z) = \frac{1}{2\pi}\int_{S_z} \boldsymbol{B}(y_1, y_2, z) \times \frac{\boldsymbol{r}}{|\boldsymbol{r}|^2}\,\mathrm{d}^2 y,$$

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Thus, for the stacked domain that we have been considering, *winding* provides the underlying topological description of relative helicity.

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For fixed boundary conditions, L is a topological invariant.

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2. If a magnetic field registers a small topological signature (winding) but has strong flux, the helicity could be large.
3. Magnetic winding and helicity, despite being intimately related, do not always behave in the same way.

Simple comparison

Theorem

Consider a linear force-free magnetic field $\boldsymbol{B}$, with force-free parameter α, in a stacked domain Ω, subject to constant magnetic diffusion η. There is no fluid flow.

$L[\boldsymbol{B}(t)] = L[\boldsymbol{B}(0)]$ *and* $H^W[\boldsymbol{B}(t)] = H^W[\boldsymbol{B}(0)]\exp(-2\alpha^2\eta t)$.

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- However, without the aid of a model, we cannot perform these calculations with solar observations.
- We have information at the photosphere - we can calculate **fluxes**.

Domain

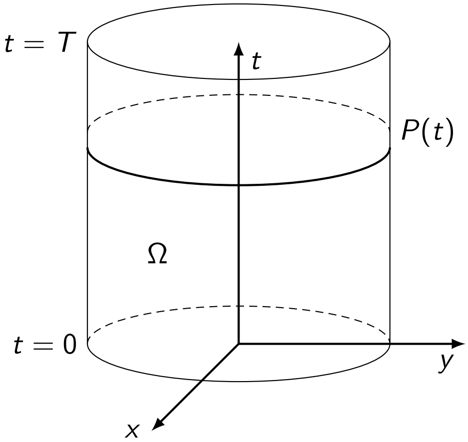


Figure: Domain of integration Ω. The plane at time t is denoted $P(t)$.

Fluxes

The (time-integrated) fluxes of relative helicity and magnetic winding are

$$H_R = -\frac{1}{2\pi}\int_0^T \int_{P\times P} \frac{\mathrm{d}}{\mathrm{d}t}\theta(\boldsymbol{x},\boldsymbol{y})B_z(\boldsymbol{x})B_z(\boldsymbol{y})\,\mathrm{d}^2x\,\mathrm{d}^2y\,\mathrm{d}t,$$

$$L_R = -\frac{1}{2\pi}\int_0^T \int_{P\times P} \frac{\mathrm{d}}{\mathrm{d}t}\theta(\boldsymbol{x},\boldsymbol{y})\sigma(\boldsymbol{x})\sigma(\boldsymbol{y})\,\mathrm{d}^2x\,\mathrm{d}^2y\,\mathrm{d}t,$$

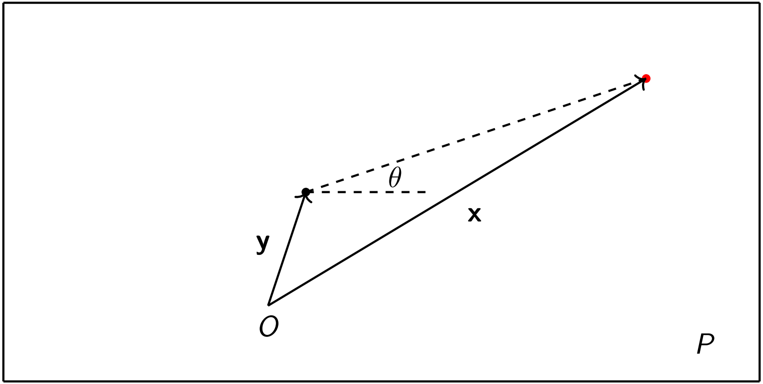
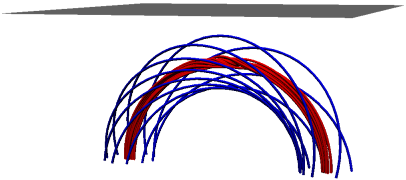
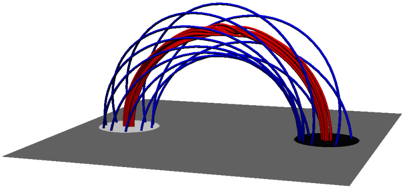
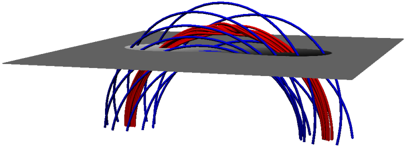


Figure: Two footpoint intersections with P shown as red and black dots. Their respective position vectors, $\mathbf{x}$ and $\mathbf{y}$, are displayed with reference to a chosen origin O. The pairwise winding angle measures the rotation of $\mathbf{x}-\mathbf{y}$ about O.

Flux emergence - toy example







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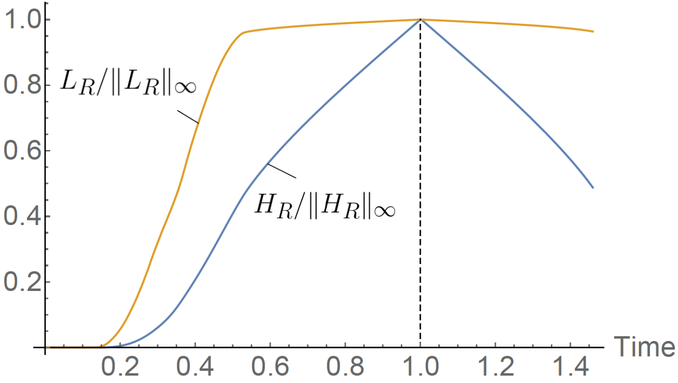
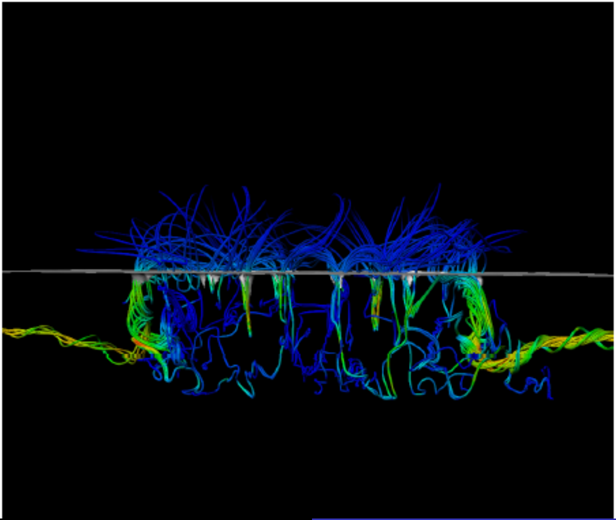
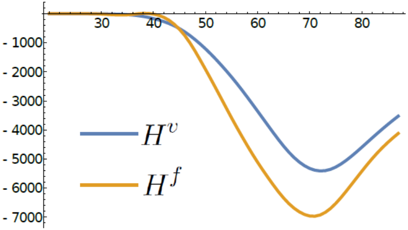


Figure: H_R and L_R input integrated in time.

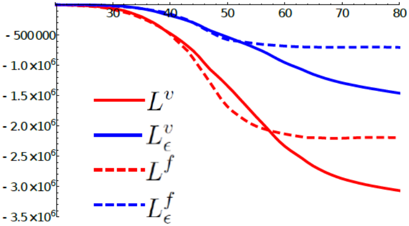
Simulation example



Simulation example



(b)



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- For classical (closed field) helicity, topology is measured by Gauss linkage.
- For relative helicity (in a stacked region), topology is measured by winding.
- Winding can be separated from helicity.
- Winding picks out topological complexity more clearly than helicity in flux emergence.

Discussion

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- Connection between winding and other helicity signatures, e.g. "non-potential helicity ratios."
- Work in progress...more theory and simulations (and observations) coming soon...

We now present a collection of papers related to the themes discussed in this talk. The papers are classified by topic. This will not represent an exhaustive list, but should act as a good starting point for further exploration.

Our work on winding and flux emergence

See preprints on
http://www.maths.gla.ac.uk/~dmactaggart/publications.html

MacTaggart, D., Prior, C., Magnetic winding – a key to unlocking topological complexity in flux emergence 2020, Journal of Physics: Conference Series, accepted

Prior, C., MacTaggart, D., Magnetic winding: what is it and what is it good for? 2020, Proceedings of the Royal Society A, accepted

MacTaggart, D., Prior, C., Helicity and winding fluxes as indicators of twisted flux emergence 2020, Geophysical and Astrophysical Fluid Dynamics, accepted

Prior, C., MacTaggart, D., Interpreting magnetic helicity flux in solar flux emergence 2019, Journal of Plasma Physics, 85, 775850201

Prior, C., MacTaggart, D., The emergence of braided magnetic fields 2016, Geophysical and Astrophysical Fluid Dynamics, 110, 432

Classical (closed field) helicity

Moffatt, H. K., The degree of knottedness of tangled vortex lines 1969, Journal of Fluid Mechanics, 35, 117

Moffatt, H. K., Ricca, R. L., Helicity and the Calugareanu invariant 1992, Proceedings of the Royal Society A, 439, 411

Laurence, P., Avellaneda, M., A Moffatt-Arnold formula for the mutual helicity of linked flux tubes 1993, Geophysical and Astrophysical Fluid Dynamics, 69, 243

MacTaggart D., Valli A., Magnetic helicity in multiply connected domains 2019, Journal of Plasma Physics, 85, 775850501

Arnold, V. I., Khesin, B. A., Topological methods in hydrodynamics 1998, vol. 125. Applied mathematical sciences. Berlin, Germany: Springer

Relative (open field) helicity

Berger M. A., Field G. B., The topological properties of magnetic helicity 1984, Journal of Fluid Mechanics, 147, 133
Finn J., Antonsen Jr T., Magnetic helicity: What is it and what is it good for? 1985, Comments on Plasma Physics and Controlled Fusion, 9, 111

For many references on the practical calculation of relative helicity, see the recent review:
Pariat E., Using Magnetic Helicity, Topology and Geometry to Investigate Complex Magnetic Fields. In MacTaggart D., Hiller A., editors, Topics in Magnetohydrodynamic Topology, Reconnection and Stability Theory (CISM Series) 2020, vol. 591, Springer

Helicity and winding

Berger M. A., Topological invariants of field lines rooted to planes 1986, Geophysical and Astrophysical Fluid Dynamics, 34, 265
Wright, A. N., Berger, M. A., The interior structure of reconnected flux tubes in a sheared plasma flow 1990, Journal of Geophysical Research, 95, 8029
Berger M. A., Prior C., The writhe of open and closed curves 2006. Journal of Physics A: Mathematical and General, 39, 8321
Prior C., Yeates A., On the helicity of open magnetic fields 2014, The Astrophysical Journal, 787, 100