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On the generation and segregation of kinetic helicity in geodynamo simulations

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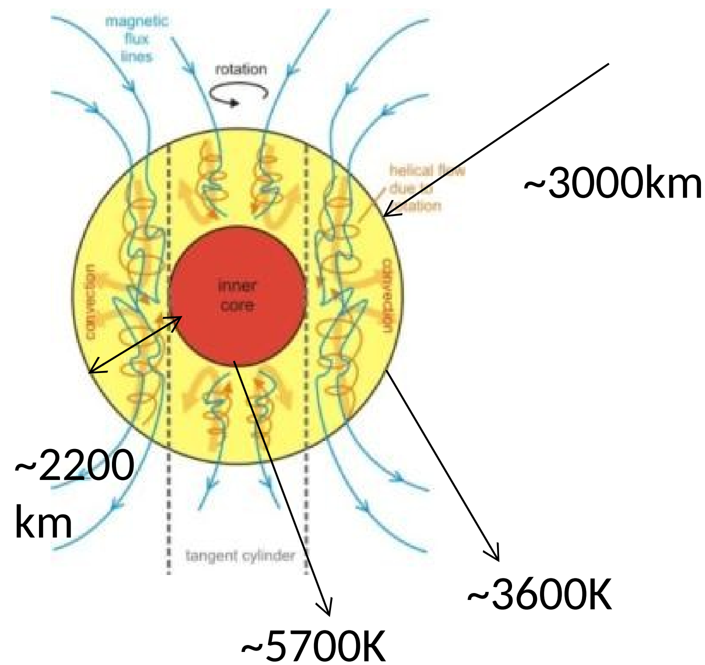
Helicity2020

Online Advanced Study Program on helicities in Astrophysics and Beyond

11 December 2020

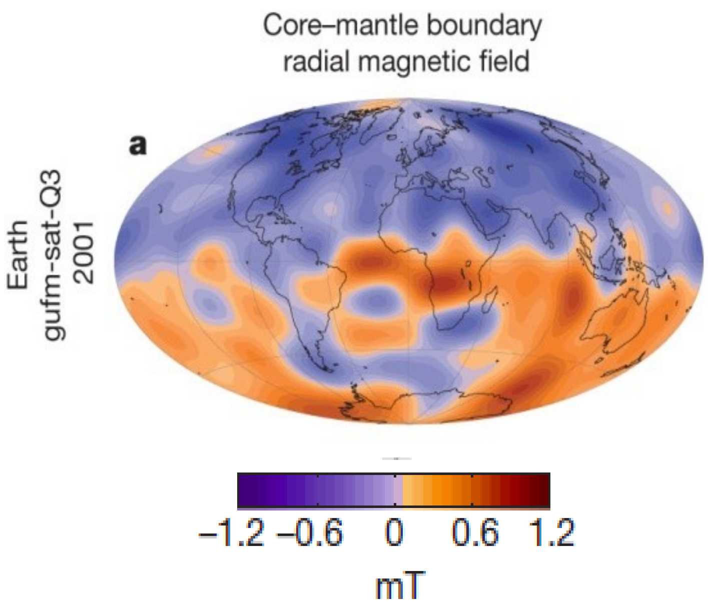
Outline

- Motivation
- Why is helicity segregation important?
- Helicity evolution equation
- Dynamo simulations
- An interesting correlation!
- DNS in a box – buoyant blobs under rotation
- Some (speculative) explanations



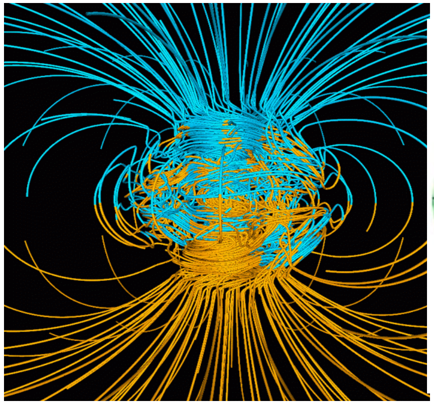
Helical columnar flow could be important for the generation and maintenance of the magnetic field !

Where does the dipolar magnetic field of the Earth come from & why is it still there?



Aubert et al 2013

Observation

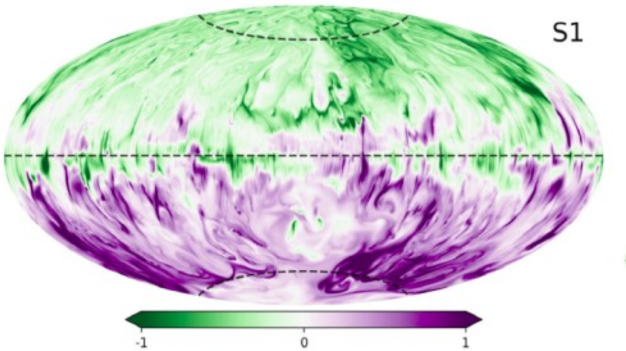


Glaztmaier& Roberts' Nature, 95; PEPI,95

49 X 32 X 64 modes

Ra = 5.7×10^{7}

E = 2×10^{-6}



Schaeffer et al. GJI, 2017

512 X 720 X 1440 modes

Ra = 1.3×10^{11}

E = 1×10^{-6}

Numerical Simulations

In the core, E ~ 10^{-15}; Ra ~ 10^{22}

(Schubert, Treatise on Geophysics)

With core radius ~ 3.5×10^{6} m; conductivity ~ 4×10^{5} S/m; magnetic diffusivity ~ 2 m^2/s

Magnetic field should have died in ~ 200, 000 years

Age of the earth ~ 4, 543, 000, 000 years

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Advance Access publication 2017 June 30
GJI Geomagnetism, rock magnetism and palaeomagnetism

doi: 10.109

Turbulent geodynamo simulations: a leap towards Earth's core

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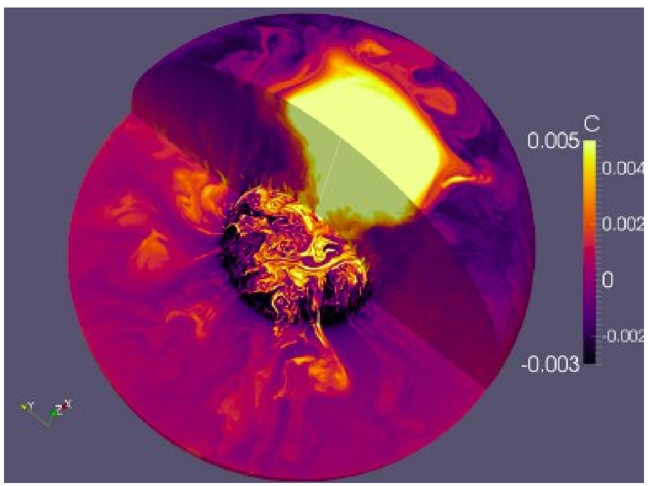
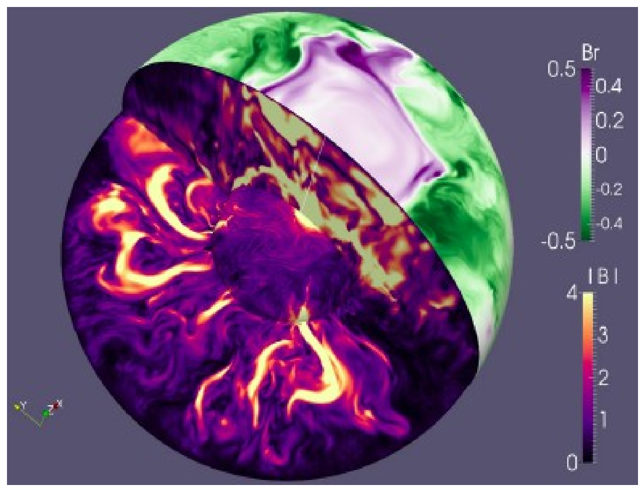
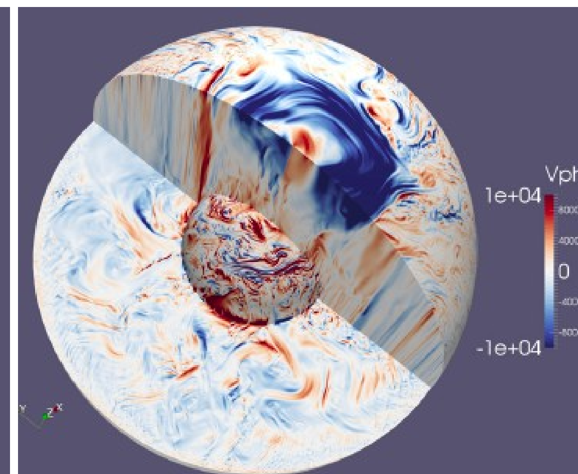


Temperature
(Co-density)

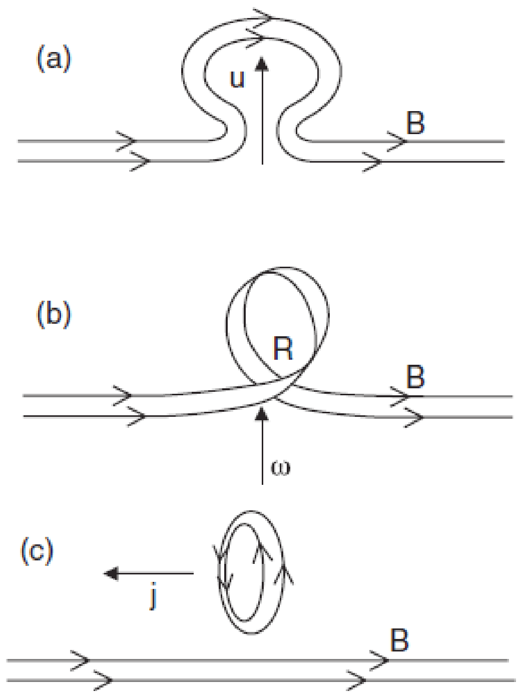


Radial comp. of **B**

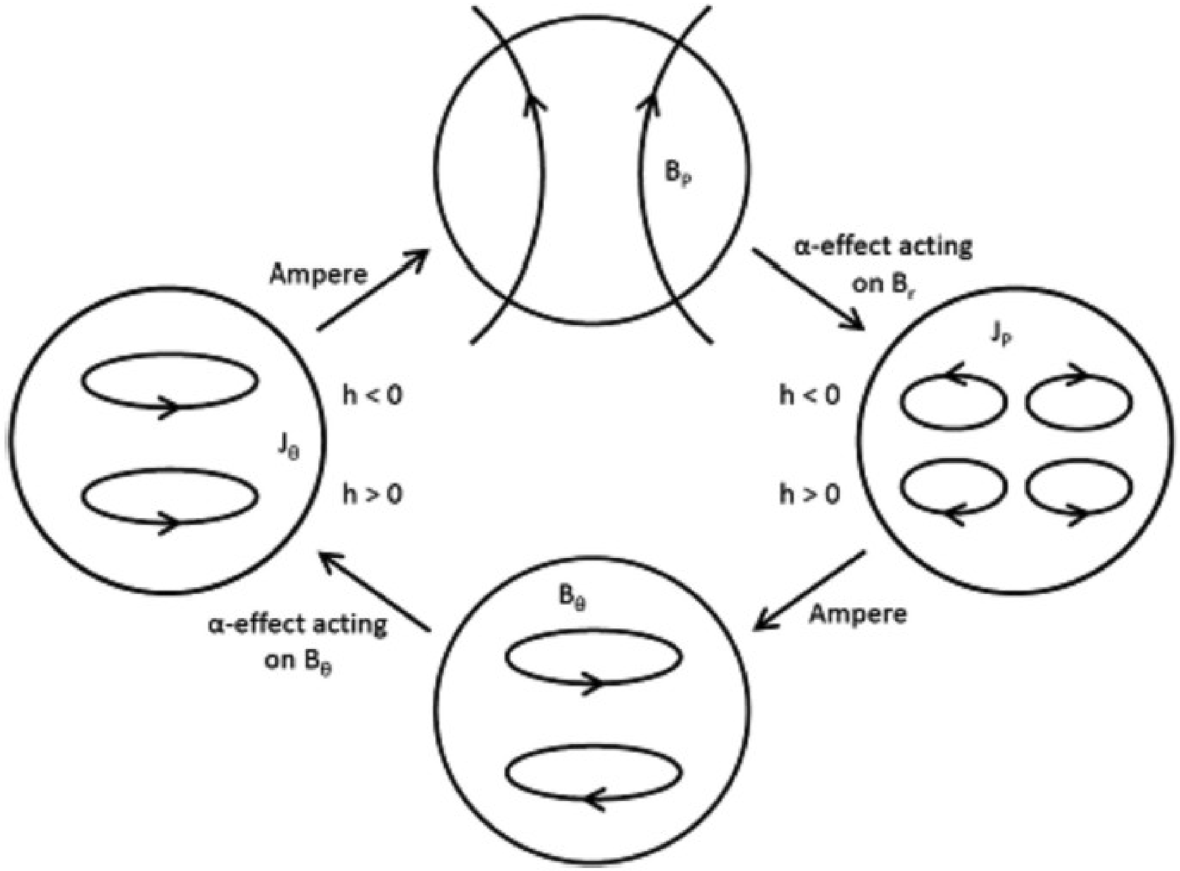


φ comp. of **u**

Parker's α-effect & α^2-dynamo



Parker's 'lift & twist'
α-effect (Parker, 1955)



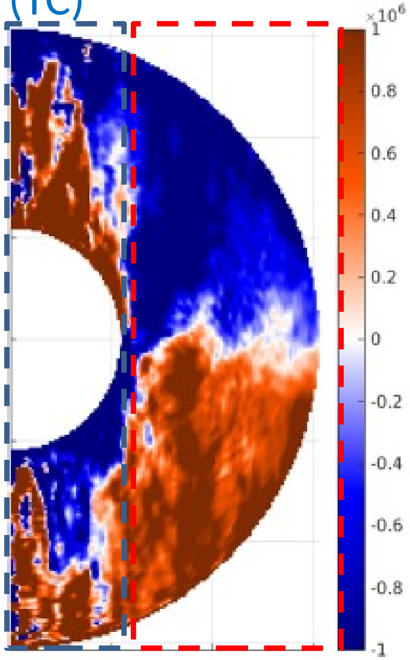
A α^2-dynamo cartoon
(Davidson, 2014)

Helicity (or helicity density)
h = **u·ω**

If the flow is helical & helicity is −ve in north +ve in the south, then we can get one component of **B** from the other & a self sustaining dynamo!

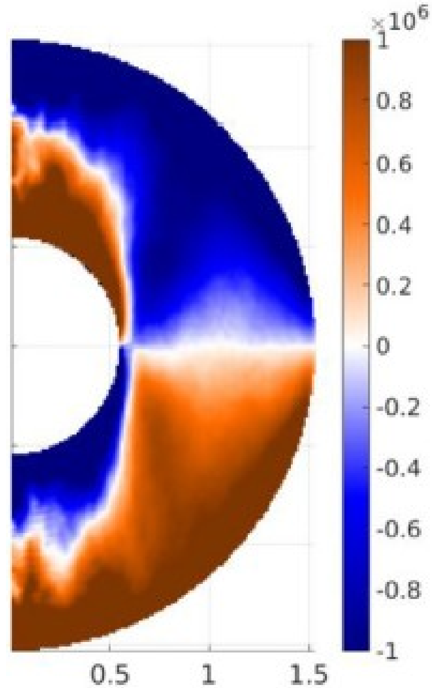
What is the helicity in dynamo simulations?

Tangent Cylinder (TC)



Intantaneous helicity avg. in φ

$$\langle h \rangle_{\phi}$$



Time-averaged helicity avg. in φ

$$\langle \overline{h} \rangle_{\phi}$$

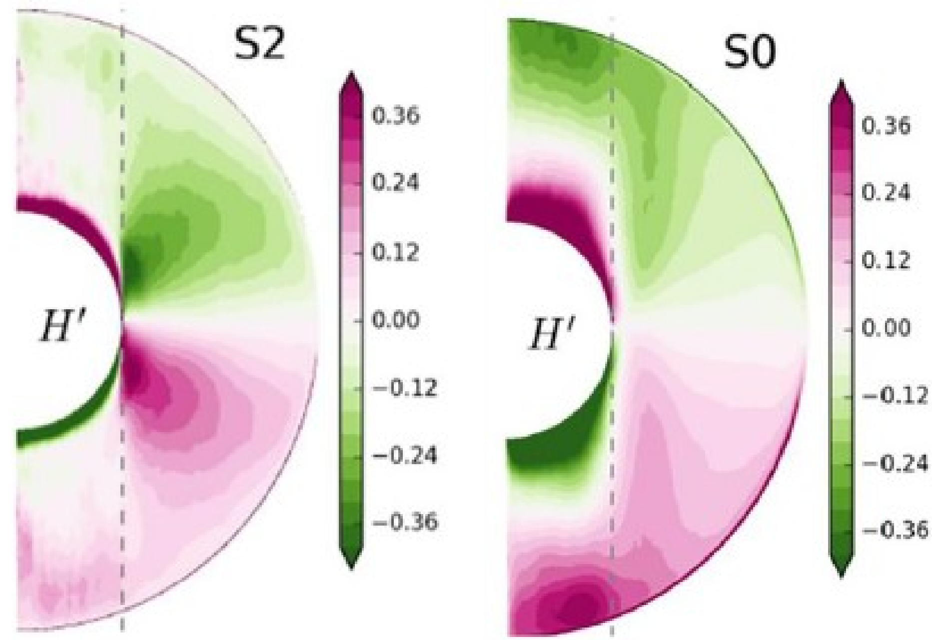


Figure 1. Relative fluctuating helicity averaged in time and in φ for two different simulations S2 and S0 (Schaeffer *et al.* 2017). Simulation parameters for S2: $Ra/Ra_c = 6310$, $E = 10^{-7}$ and S0: $Ra/Ra_c = 4879$, $E = 10^{-5}$. Ra is the Rayleigh number (defined in Section 3), which equals Ra_c at the onset of convection. Fixed heat flux boundary condition was used in these simulations.

$$\overline{u' \cdot \omega'} / \overline{|u'|}\,\overline{|\omega'|}_{\varphi}$$

$$u'(x,\ t) = u(x,t) - \bar{u}(x)$$

Sources of helicity according to literature

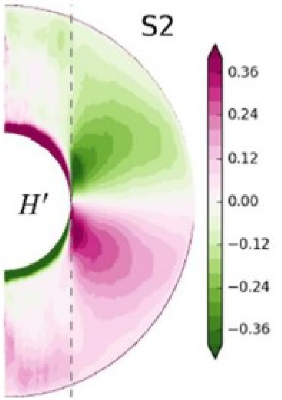
According to Busse (1975, 1976), three possible sources of helicity outside the tangent cylinder could be

(a) Ekman pumping - this could be important only at large Ekman number & only near the boundaries.

(b) the spherical boundary (β-effect) -this only leads to intensification of helicity

(c) spatial variations of the Lorentz force.

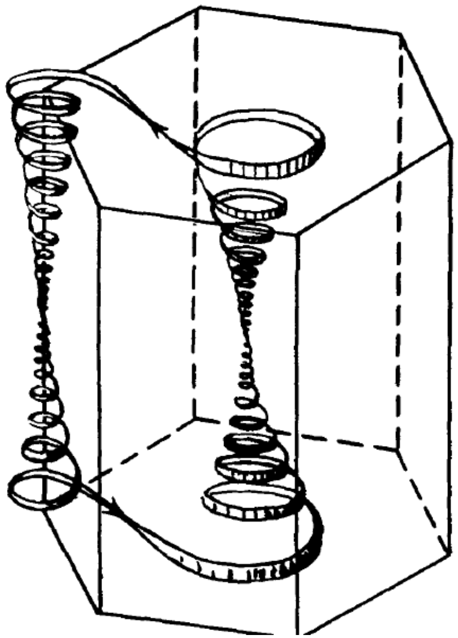
- this is a certainly a source but, again, only leads to a local intensification of helicity (Sreenivasan & Jones, 2011)



But, what leads to the segregation of helicity (-ve above, + below) in the simulations?

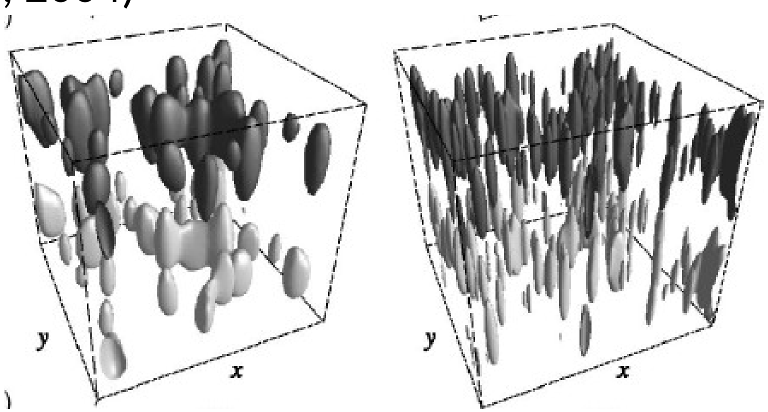
This is observed in simulations with free-slip B.C.s (ruling out Ekman pumping) and in non-magnetic simulations (ruling out Lorentz force)!

There is less debate for helicity in rotating RBC/ inside TC where Ω // g

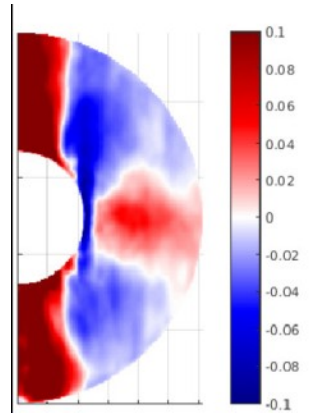


Veronis (JFM, 1959)

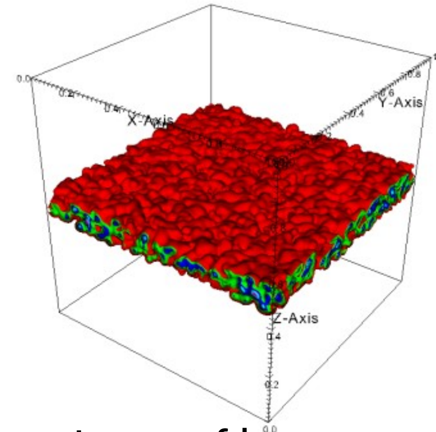
Stellmach & Hansen (PRE, 2004)



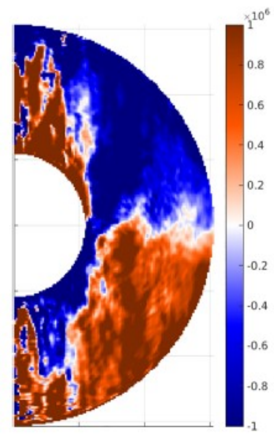
A “radical” idea proposed by Davidson (GJI, 2014)



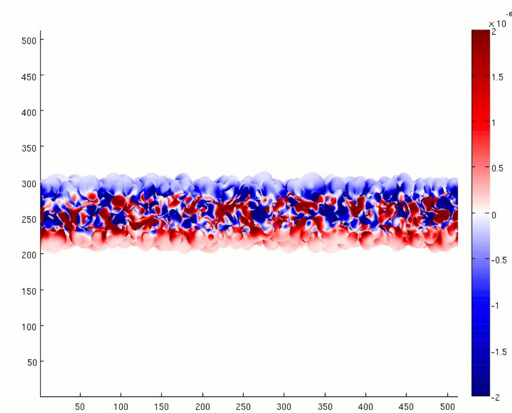
Temperature perturbations avg. in φ



Layer of buoyancy with **Ω** perp. to **g**



$\langle h \rangle_\phi$



Kinetic energy coloured with *h*

Davidson & Ranjan (GJI, 2015)

Inertial waves & helicity

$$\frac{\partial^2}{\partial t^2}\nabla^2 \boldsymbol{u} + 4\left(\boldsymbol{\Omega}\cdot\nabla\right)^2 \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla\tilde{p} - 2\boldsymbol{\Omega}\times\boldsymbol{u}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = 2(\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{u}$$

$$\boldsymbol{u} = \hat{\boldsymbol{u}}\exp i(\boldsymbol{k}\cdot\boldsymbol{x} - \varpi t)$$

$$\varpi = \pm 2\frac{(\mathbf{k}\cdot\boldsymbol{\Omega})}{\mathbf{k}} = \pm 2\Omega\cos\theta$$

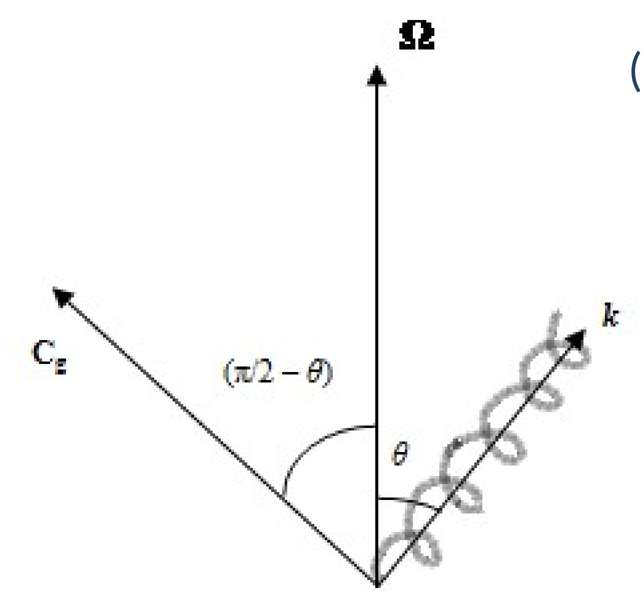
$$\varpi/\Omega = (-2,2)$$

$$\theta \approx 90^\circ \quad \varpi \approx 0$$

$$\mathbf{C_g} = \frac{\partial\varpi}{\partial k} = \pm\frac{2\Omega}{k}\left[\hat{\boldsymbol{e}}_\Omega - \cos\theta\,\hat{\boldsymbol{e}}_{\boldsymbol{k}}\right]$$

$$\mathbf{C_g} = \pm\frac{2\Omega}{k}\hat{\boldsymbol{e}}_\Omega$$

(Rate at which energy propagates)

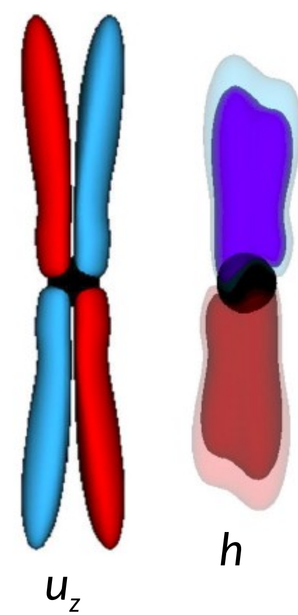


$$\hat{\boldsymbol{\omega}} = i\boldsymbol{k}\times\hat{\boldsymbol{u}} = \mp k\,\hat{\boldsymbol{u}}$$

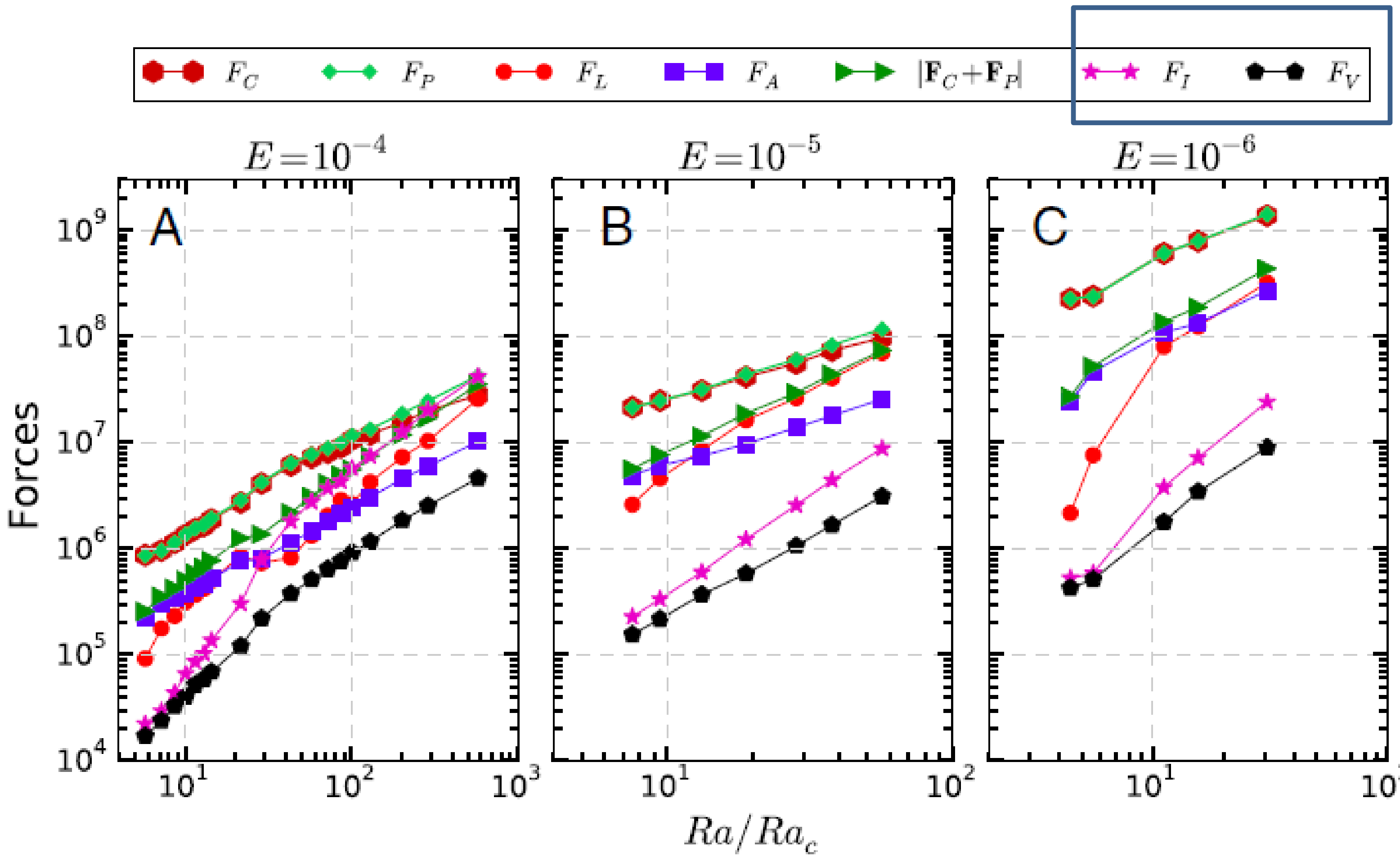
$$h = \boldsymbol{u}\cdot\boldsymbol{\omega} = \mp k\left|\hat{\boldsymbol{u}}\right|^2$$

Note Parallel velocity
& vorticity amplitudes

-ve above
+ve below
(Moffatt, 1970)



MAC Balance in geodynamo simulations



Yadav et al. PNAS, 2016

Helicity budget equation

$2\mathbf{u}\times\boldsymbol{\Omega}$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{\nabla p^*}{\rho} + 2(\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{a} - \alpha T \mathbf{g} + \frac{\boldsymbol{J}\times\boldsymbol{B}}{\rho} + \nu\nabla^2\boldsymbol{u},$$

$$(\boldsymbol{u} = \nabla\times\boldsymbol{a},\ \nabla\cdot\boldsymbol{a} = 0),$$

$$p^* = p - (\rho/2)(\boldsymbol{\Omega}\times\boldsymbol{x})^2 + 2\rho(\boldsymbol{a}\cdot\boldsymbol{\Omega}).$$

$$\frac{\partial \omega}{\partial t} = 2(\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{u} - \alpha\nabla T\times\mathbf{g} + \frac{\nabla\times(\boldsymbol{J}\times\boldsymbol{B})}{\rho} + \nu\nabla^2\omega.$$

$$\frac{\partial h}{\partial t} + \nabla\cdot\boldsymbol{F} = H_T + H_B + H_\nu,$$

where

$$\boldsymbol{F} = -2(\boldsymbol{u}^2)\boldsymbol{\Omega} - 2\boldsymbol{u}\times(\boldsymbol{\Omega}\cdot\nabla\boldsymbol{a}) + \frac{p^*\omega}{\rho},$$

and,

$$H_T = -\alpha[\boldsymbol{u}\cdot(\nabla T\times\mathbf{g}) + \omega\cdot(T\mathbf{g})],$$

$$H_B = \frac{1}{\rho}[\omega\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \boldsymbol{u}\cdot(\nabla\times(\boldsymbol{J}\times\boldsymbol{B})],$$

$$H_\nu = \nu\left(\omega\cdot\nabla^2\boldsymbol{u} + \boldsymbol{u}\cdot\nabla^2\omega\right).$$

F **– Flux of helicity**

H_T, H_B, H_ν **"sources" of helicity**

$$\nabla^2 p^* = -\rho\nabla\cdot(\alpha T\mathbf{g}),\ \boldsymbol{F} \approx -(2\boldsymbol{u}^2)\boldsymbol{\Omega}.$$

$$\langle\bar{\Sigma}\rangle_\varphi = \langle -\overline{\nabla\cdot\boldsymbol{F}}_\Omega + \bar{H}_T + \bar{H}_B + \bar{H}_\nu\rangle_\varphi.$$

Geodynamo Simulation using spherical DNS code

(Dimensionless) governing equations & parameters

$$\frac{\partial \mathbf{u}}{\partial t'} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p - \frac{2}{E}\hat{\mathbf{e}}_z \times \mathbf{u} + \frac{Ra}{Pr}T\frac{\mathbf{r}}{r_o} + \frac{1}{EPm}(\nabla\times\mathbf{B})\times\mathbf{B} + \nabla^2\mathbf{u}$$

Magnetic Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t'} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B}$$

T is the temperature fluctuation above the adiabatic reference!

$$\frac{\partial T}{\partial t'} + (\mathbf{u}\cdot\nabla)T = \frac{1}{Pr}\nabla^2 T$$

$$\nabla\cdot\mathbf{u} = \nabla\cdot\mathbf{B} = 0$$

$$t' = t\nu / D^2$$

$$x = x / D$$

$$u = u\, D/\nu$$

$$T = T / \Delta T$$

$$B = B/(\mu_o \lambda \rho\, \Omega)^{1/2}$$

$$Ra = \frac{\alpha g_o \Delta T D^3}{\kappa\nu} \qquad E = \frac{\nu}{\Omega D^2}$$

Case	Resolution (N_r, N_θ, N_φ)	Ra (x10^8)	Ra/Ra_c	E (x10^{-5})
S1	(81,192,384)	1.2	42.4	3
S1H	(81,192,384)	1.2	42.4	3
S2	(73,256,512)	3	28.4	1

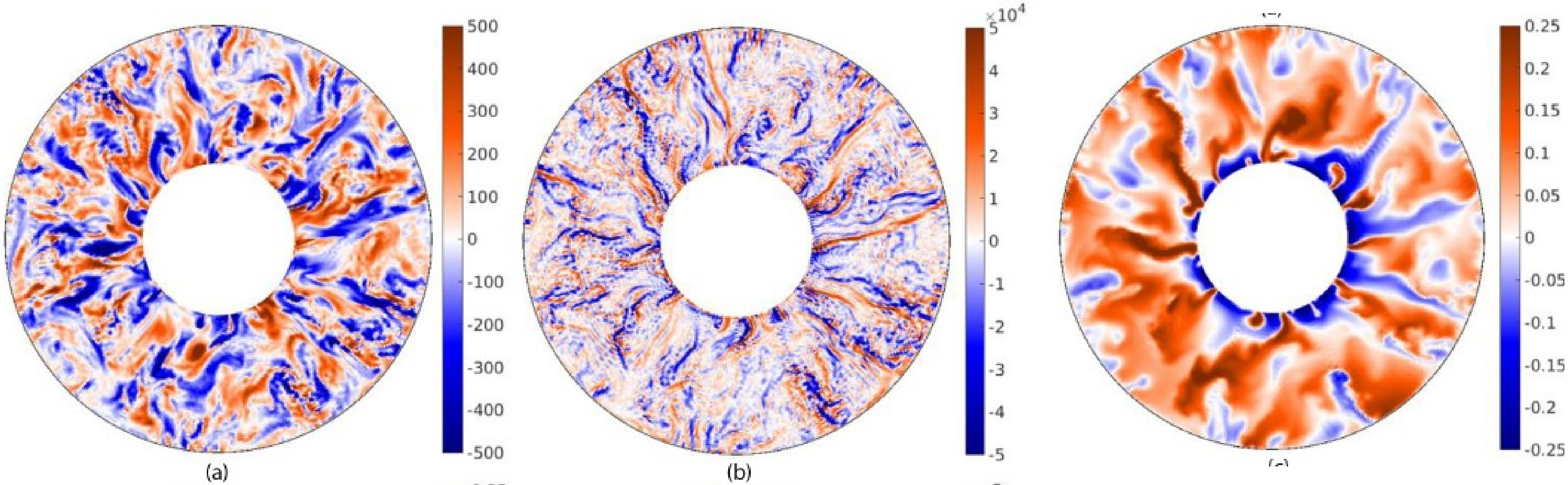
The DNS code **MagIC** solves the pseudo-spectral versions of these equations, **using spherical harmonics in (θ,*φ*) and Chebyshev polynomials in *r*.**

Boundary Cond. – No Slip, Fixed temperatures on ICB,CMB; electrically insulated IC

S1/S2 are run for ~ 0.1/0.064 magnetic diffusion time until they reach equilibrium & then re-started and run for ~ 12/17 eddy turnover time with output at small intervals

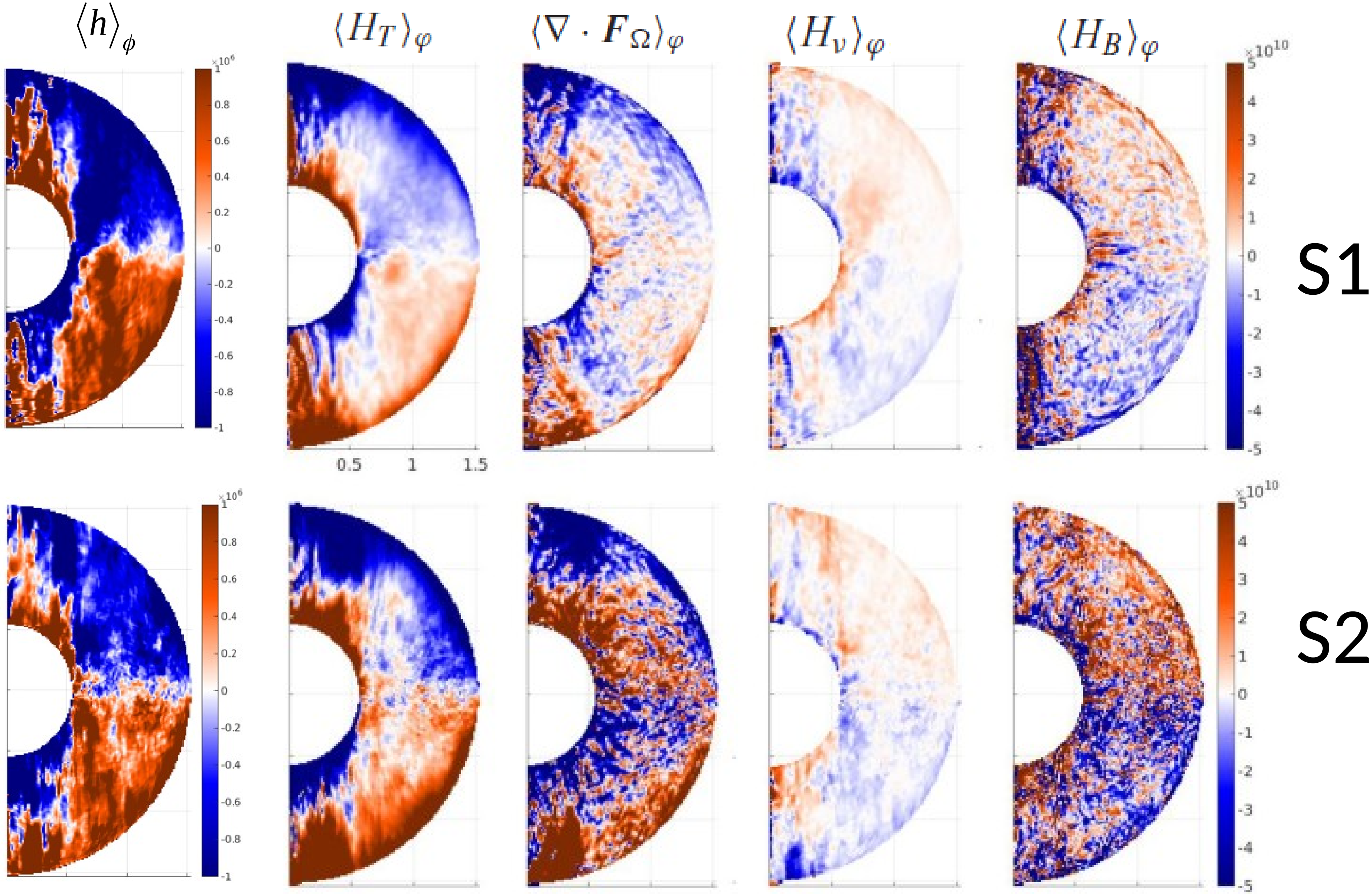
Results

For S2 Equatorial cuts of vertical velocity u_z, vertical vorticity, and temperature perturbations T

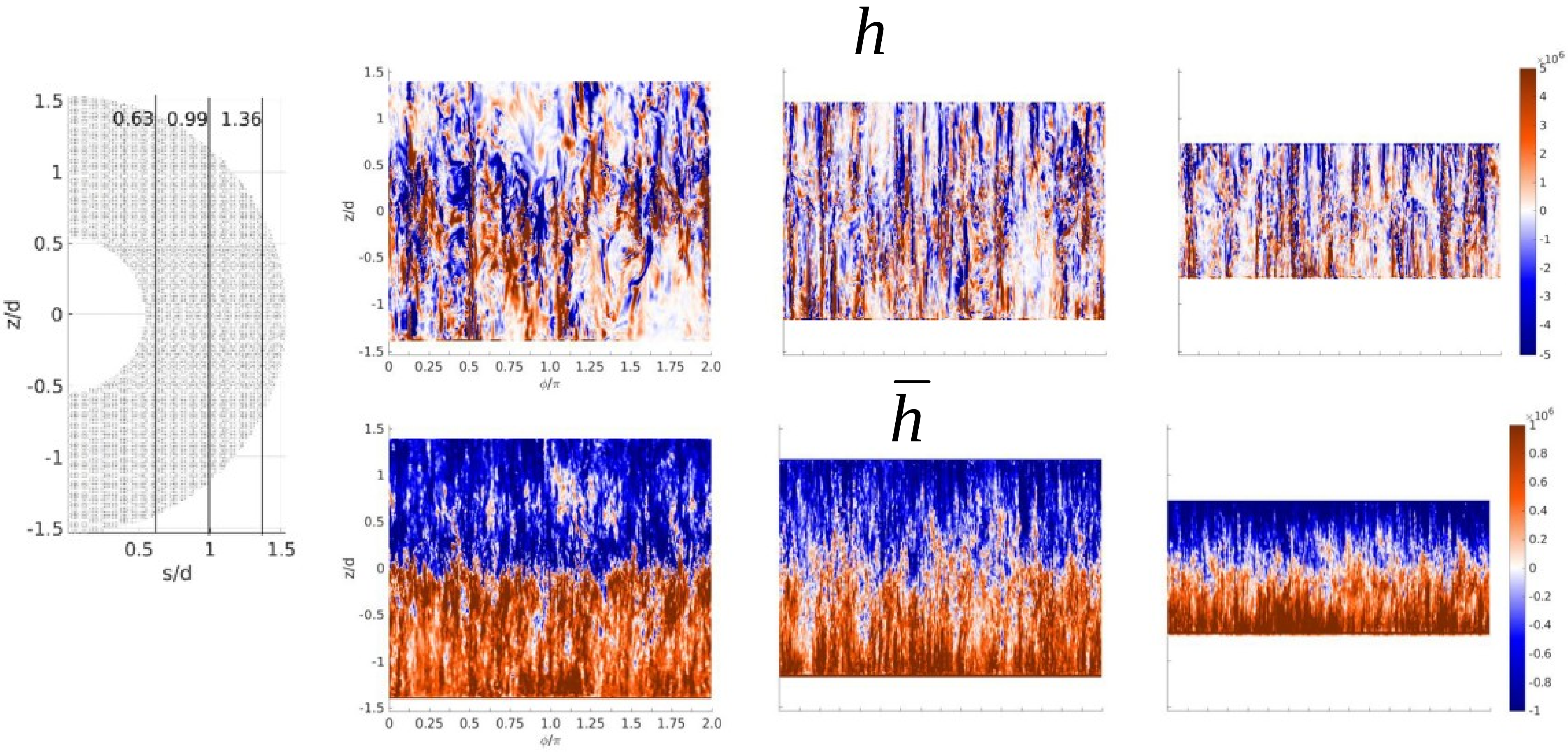


Instantaneous helicity flux & sources

$$\frac{\partial h}{\partial t} + \nabla \cdot \boldsymbol{F} = H_T + H_B + H_\nu,$$

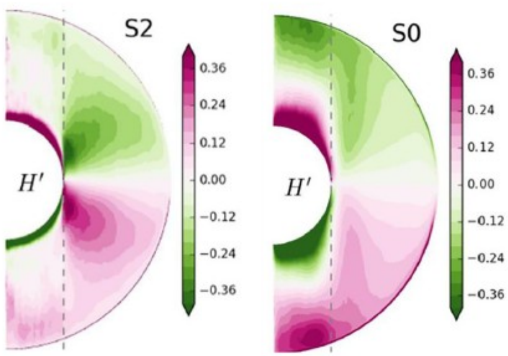


Helicity on φ-z planes for S2



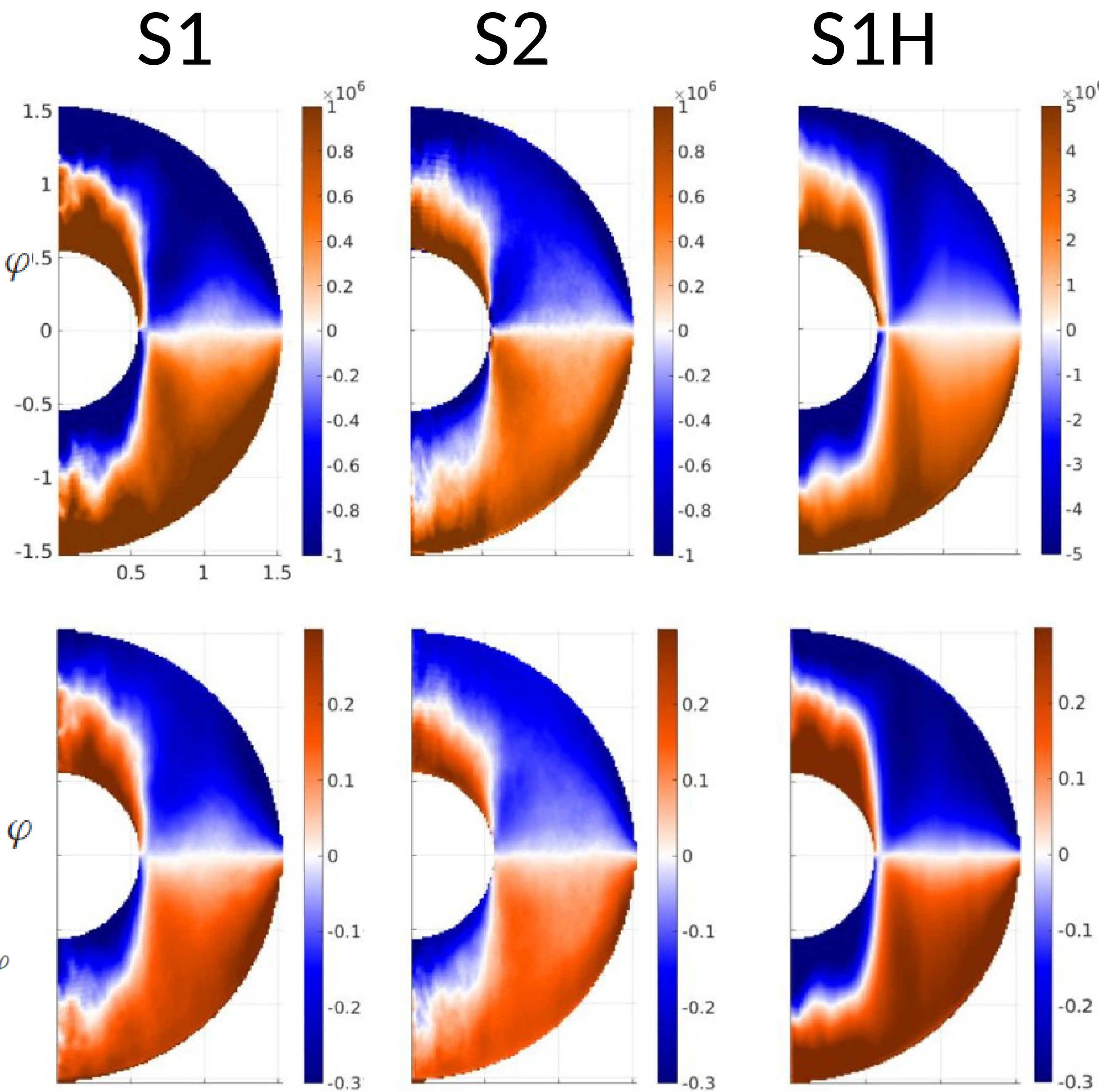
Time-averaged plots of helicity and fluctuating relative helicity

RECALL

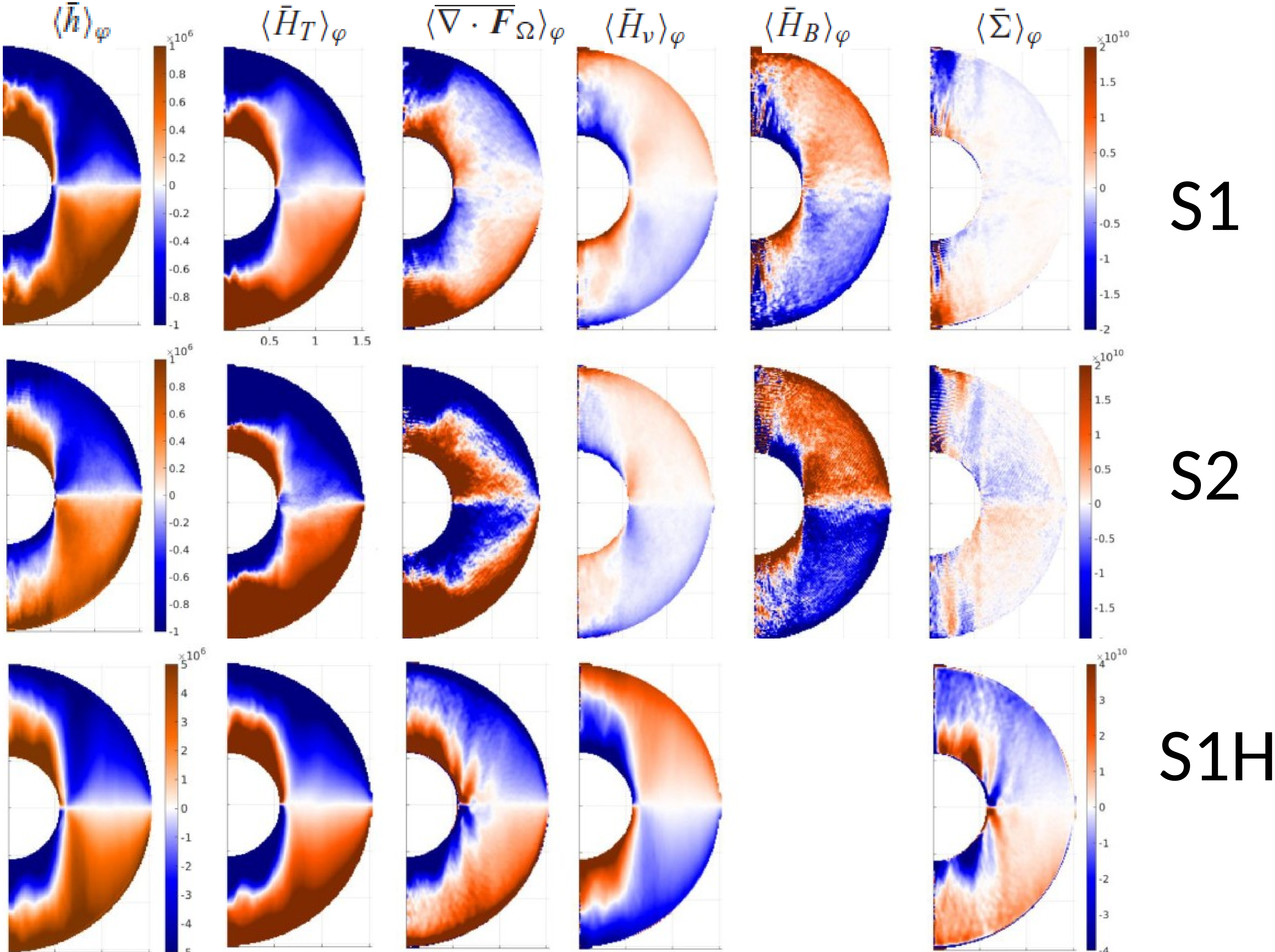


$\langle \bar{h} \rangle_\varphi$

$\langle \overline{h'}_r \rangle_\varphi = \overline{\boldsymbol{u}' \cdot \boldsymbol{\omega}'} / |\overline{\boldsymbol{u}'}||\overline{\boldsymbol{\omega}'}|_\varphi$



Time-averaged helicity flux, sources and residual



Main observations & some explanations

- H_T is strongly correlated with h
- H_B are H_ν are oppositely correlated with H_T ,h
- H_ν is small, others have comparable magnitudes
- The flux is relatively more complicated in the dynamos
- *At locations where h is negative, a positive value of* $\boldsymbol{\omega}\cdot(\boldsymbol{J}\times\boldsymbol{B})$ *means negative* $\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})$ *which is expected since magnetic energy grows at the expense of kin. energy*
- *For a helical wave* $H_\nu = \nu(\omega \cdot \nabla^2 u + u \cdot \nabla^2 \omega) \sim -2\nu k^2 h.$

$$\frac{\partial h}{\partial t} + \nabla \cdot \boldsymbol{F} = H_T + H_B + H_\nu,$$

where

$$\boldsymbol{F} = -2\left(\boldsymbol{u}^2\right)\boldsymbol{\Omega} - 2\boldsymbol{u} \times (\boldsymbol{\Omega}\cdot\nabla a) + \frac{p^*\omega}{\rho},$$

and,

$$H_T = -\alpha\left[\boldsymbol{u}\cdot(\nabla T \times \mathrm{g}) + \omega\cdot(T\mathrm{g})\right],$$

$$H_B = \frac{1}{\rho}\left[\omega\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \boldsymbol{u}\cdot(\nabla\times(\boldsymbol{J}\times\boldsymbol{B})\right],$$

$$H_\nu = \nu\left(\omega\cdot\nabla^2\boldsymbol{u} + \boldsymbol{u}\cdot\nabla^2\omega\right).$$

Why is H_T correlated with h?

Let us look at H_T in more detail!

$$H_T = -\alpha[u \cdot (\nabla T \times \mathrm{g}) + \omega \cdot (T\mathrm{g})].$$

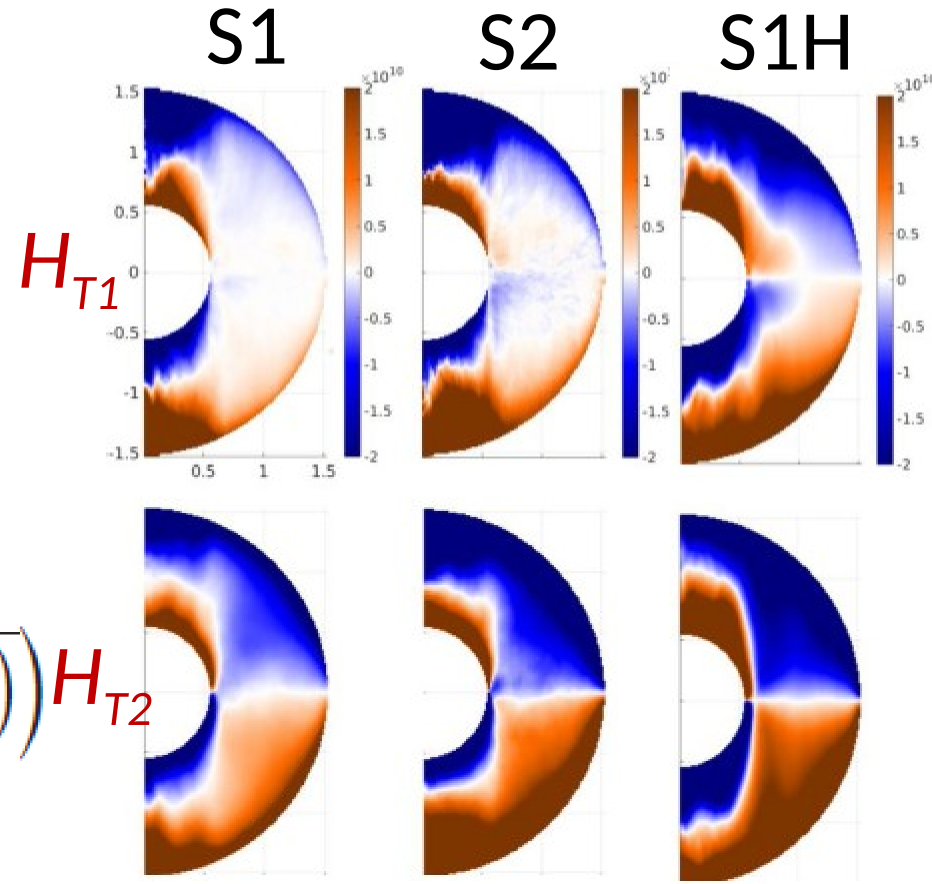
$$H_T = H_{T1} + H_{T2},$$

$$H_{T1} = \nabla \cdot (u \times (-\alpha T \mathrm{g}))$$

$$H_{T2} = -2\alpha\, u \cdot (\nabla T \times \mathrm{g}).$$

In cylindrical co-ordinates

$$\bar{H}_{T2} = \frac{2\alpha g_0}{r_0} \overline{\left((z u_s - s u_z) \frac{\partial T}{s \partial \varphi} + u_\varphi \left(s \frac{\partial T}{\partial z} - z \frac{\partial T}{\partial s} \right) \right)}$$



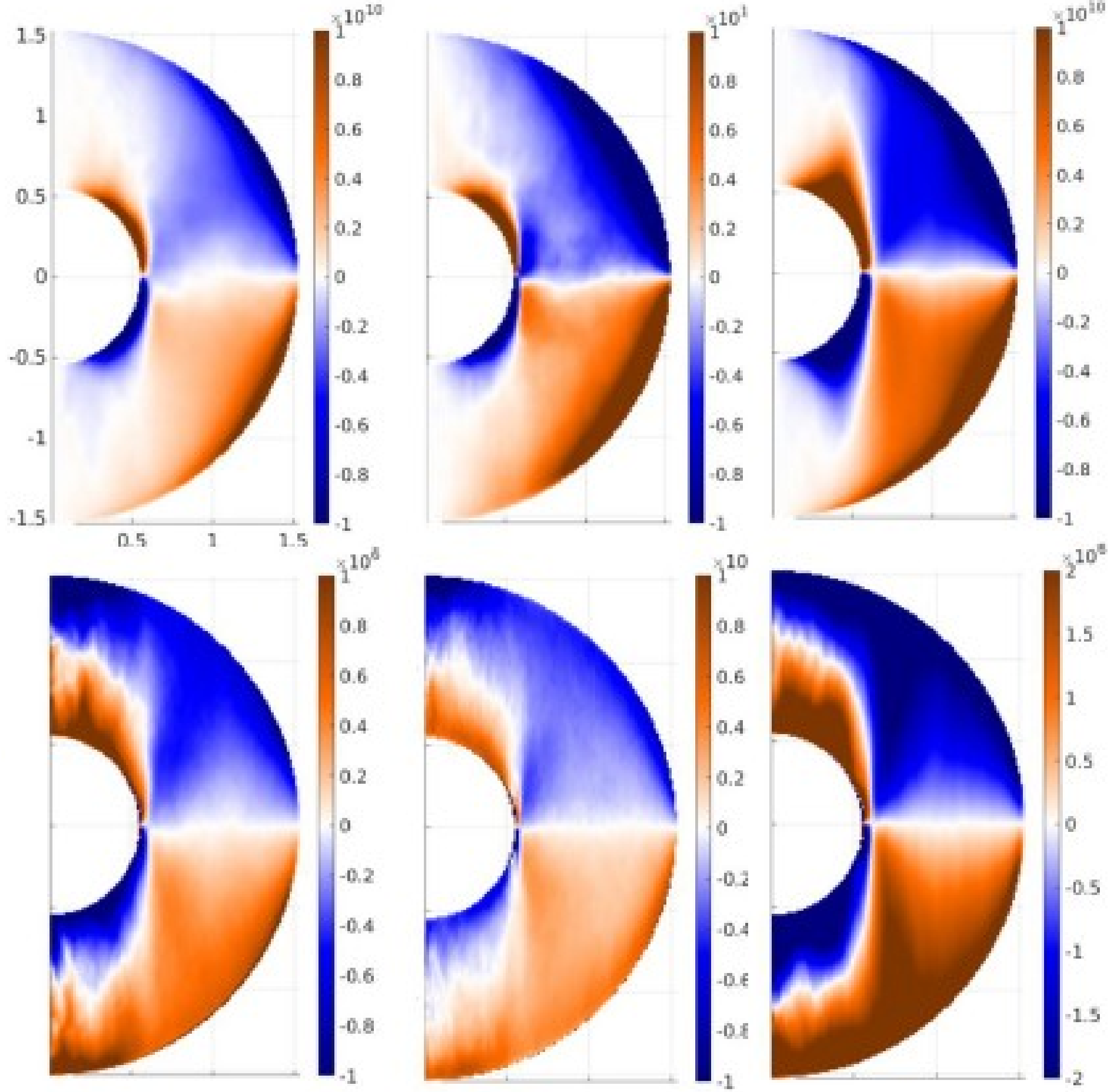
Major contribution to H_T comes from

$$- u_z\, \partial T / \partial \varphi$$

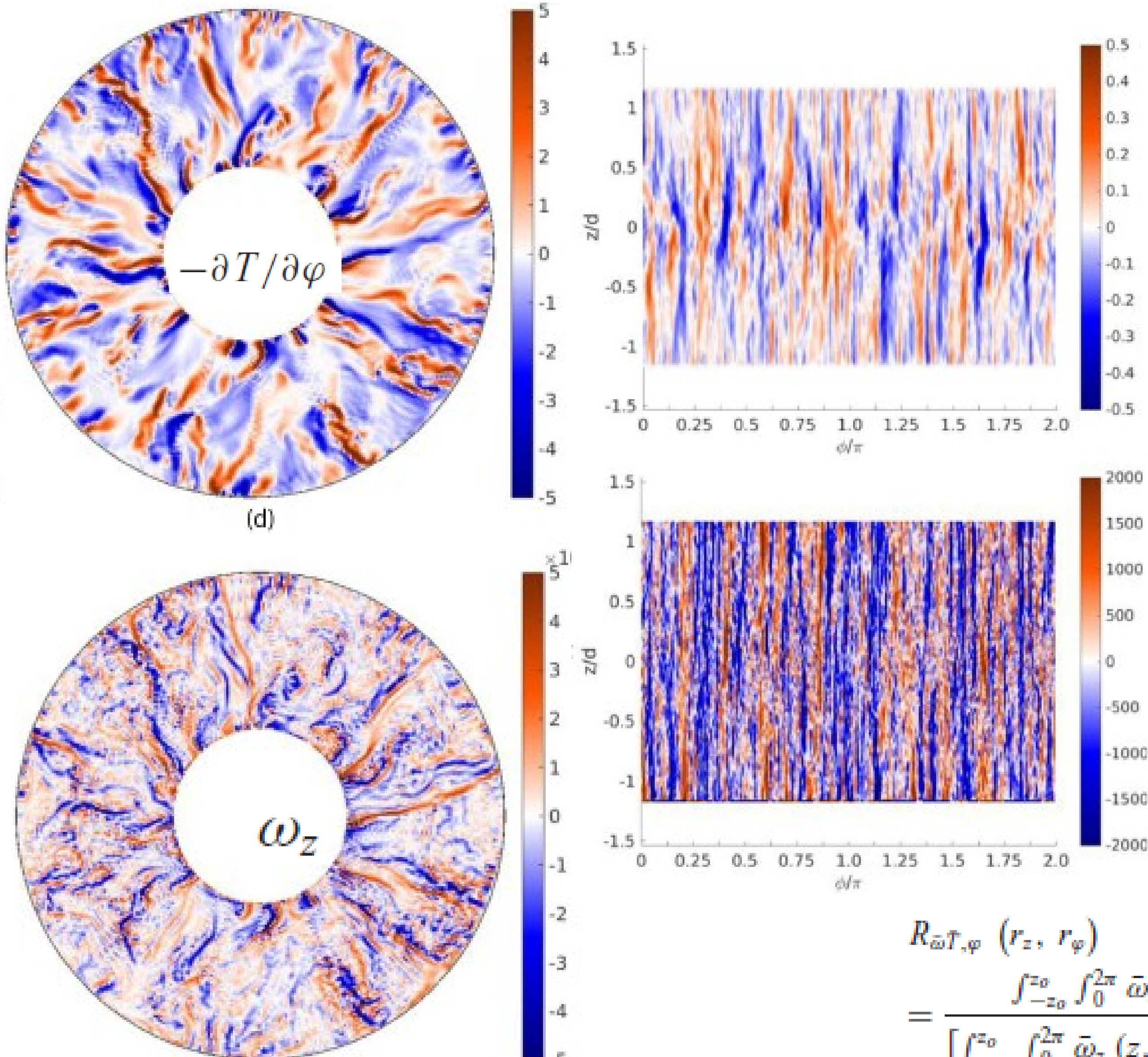
S1 S2 S1H

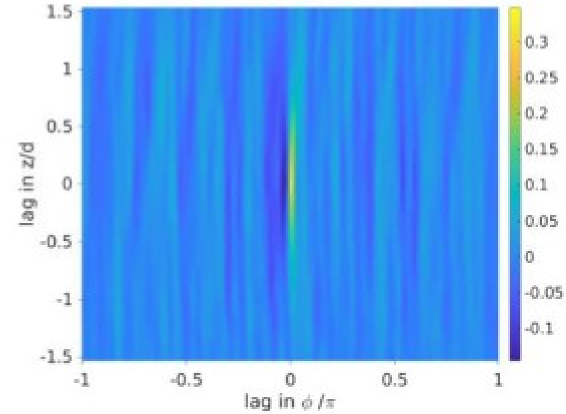
$\langle(\alpha g_0/r_0)(\overline{-u_z \partial T/\partial \varphi}\rangle_\varphi$

$\langle\overline{u_z \omega_z}\rangle_\varphi$



If ω_z and - $\partial T/\partial \varphi$ are positively correlated then we have an explanation why *h* correlates with H_T





$$R_{\bar{\omega}\bar{T},\varphi}\left(r_z,\ r_\varphi\right) = \frac{\int_{-z_o}^{z_o}\int_0^{2\pi}\bar{\omega}_z\left(z,\varphi\right)\ \bar{T}_\varphi\left(z+r_z,\varphi+r_\varphi\right)\mathrm{d}\varphi\mathrm{d}z}{\left[\int_{-z_o}^{z_o}\int_0^{2\pi}\bar{\omega}_z\left(z,\varphi\right)^2\mathrm{d}\varphi\mathrm{d}z\ \int_{-z_o}^{z_o}\int_0^{2\pi}\bar{T}_\varphi\left(z,\varphi\right)^2\mathrm{d}\varphi\mathrm{d}z\right]^{1/2}},$$

ω_z and $-\partial T/\partial\varphi$ are indeed positively correlated.

But, we do not know why!

DNS of a buoyant blob at small *Ro*

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla\tilde{p} + \nu\frac{\partial^2 \mathbf{u}}{\partial x^2} - 2\mathbf{\Omega}\times\mathbf{u} + \vartheta\mathbf{g}$$

$$\vartheta = -\vartheta_0 \exp(-(x^2+y^2)/\delta^2)\exp(-z^2/(\alpha\delta)^2)$$

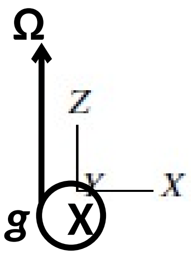
$$\alpha = 2$$

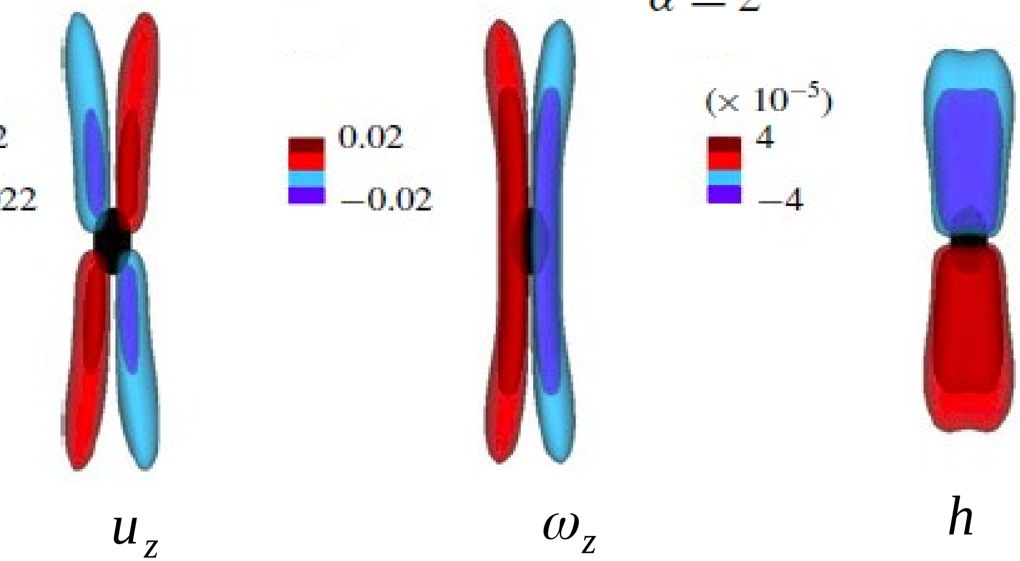
$$\boldsymbol{g} = -g\hat{\boldsymbol{e}}_y \quad \boldsymbol{\Omega} = \Omega\hat{\boldsymbol{e}}_z$$

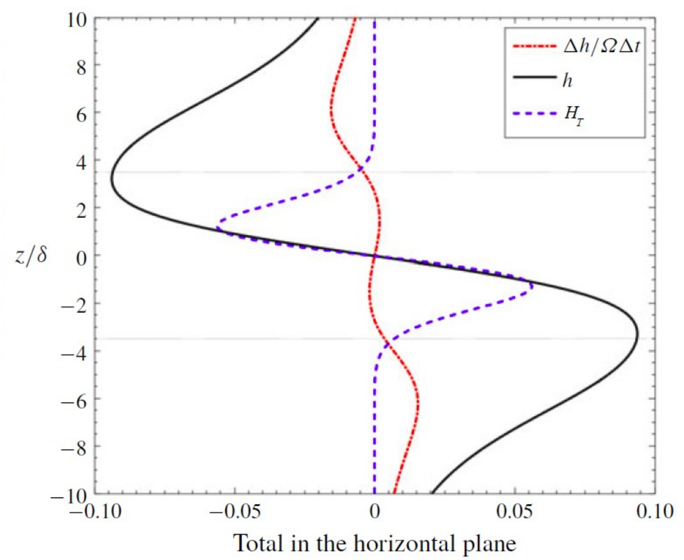
$$Ro = \vartheta_0 g/2\Omega^2\delta = 0.01$$

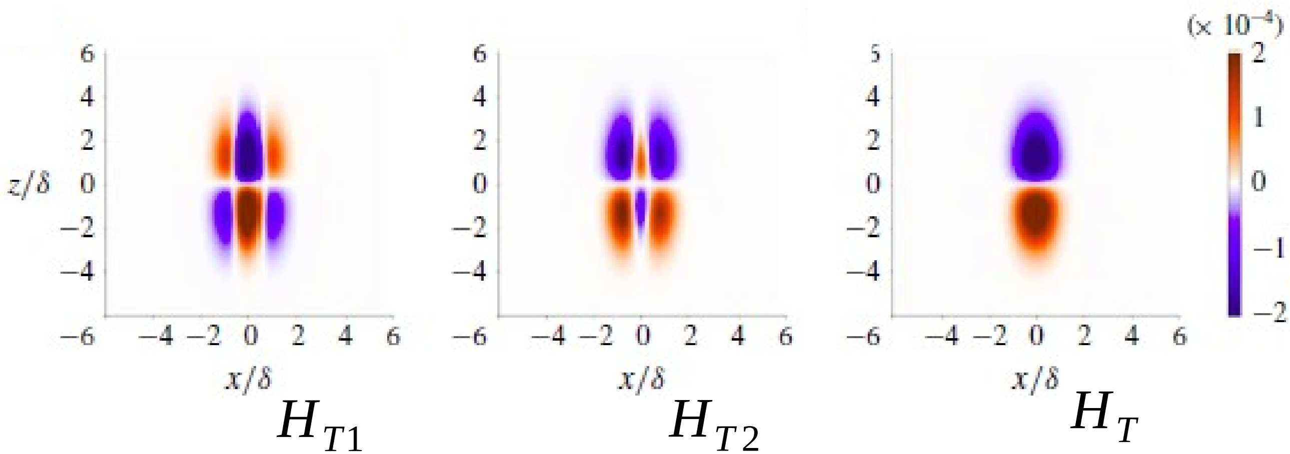
$$Ek = \nu/\Omega\delta^2 = 0.029$$

$\Omega t = 8$







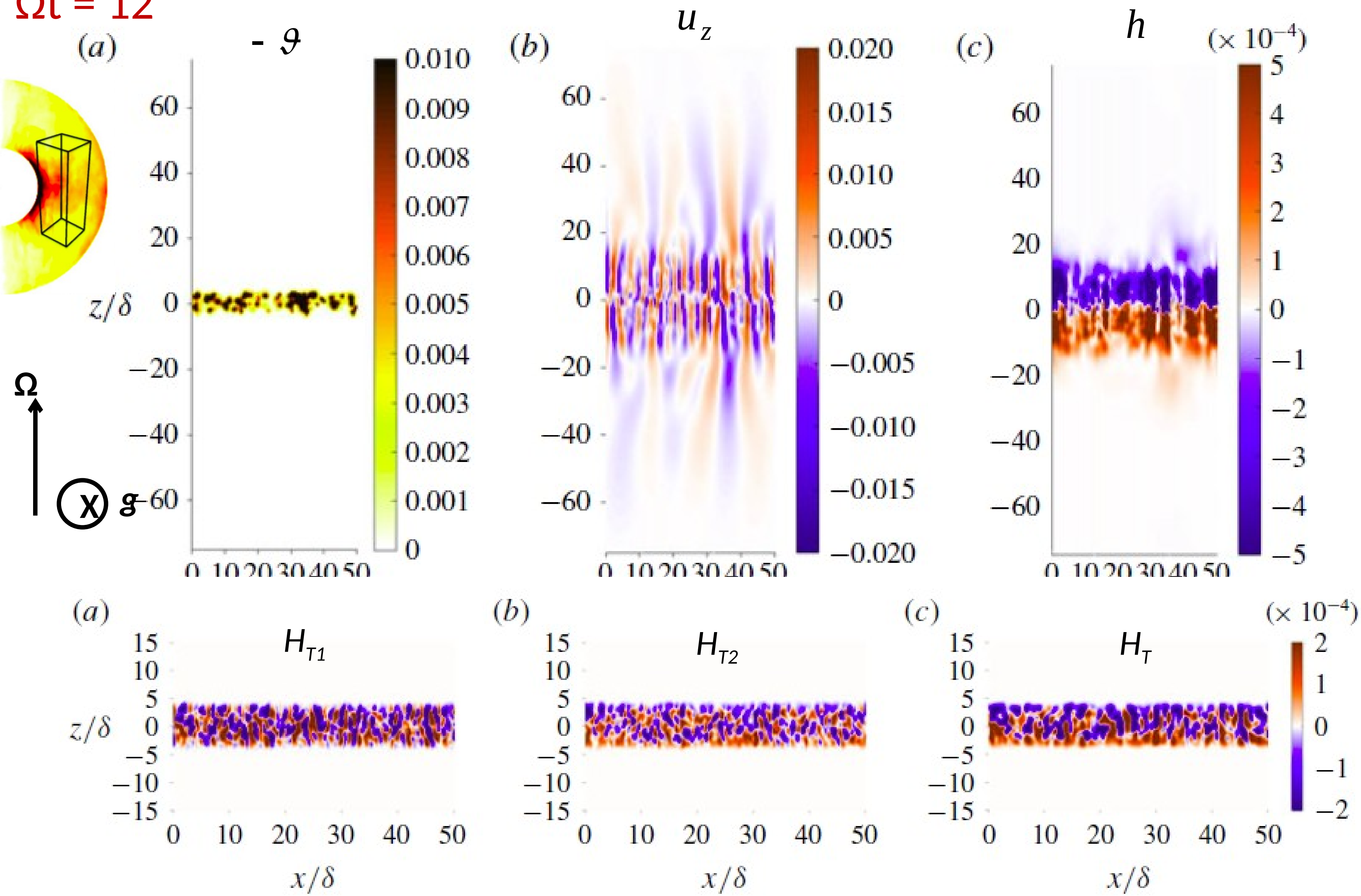


$$\frac{\partial h}{\partial t} = -\nabla\cdot\mathbf{F} + H_T$$

Once again *h* is correlated with H_T

What if we have multiple blobs randomly distributed in the centre of a box?

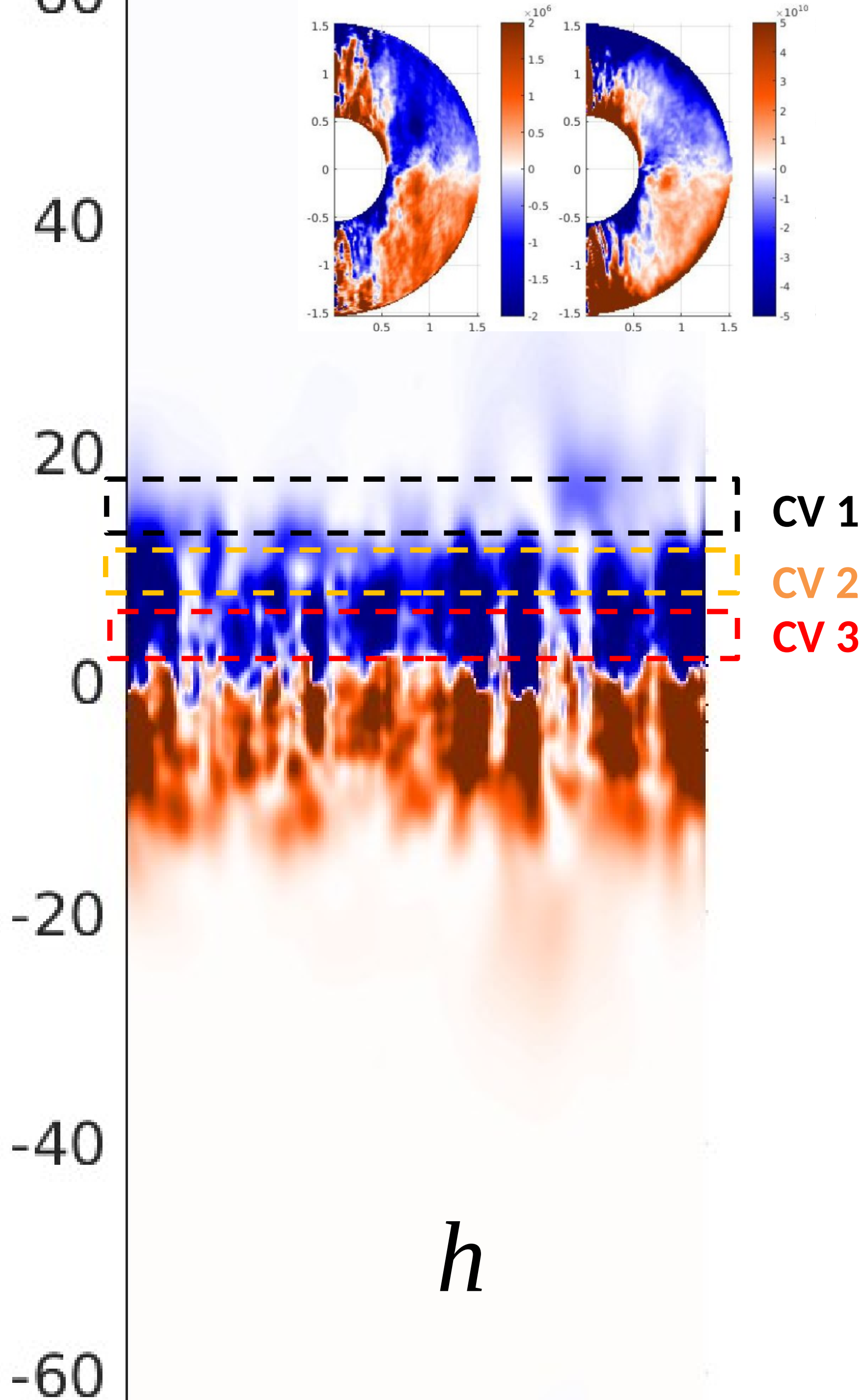
Ωt = 12

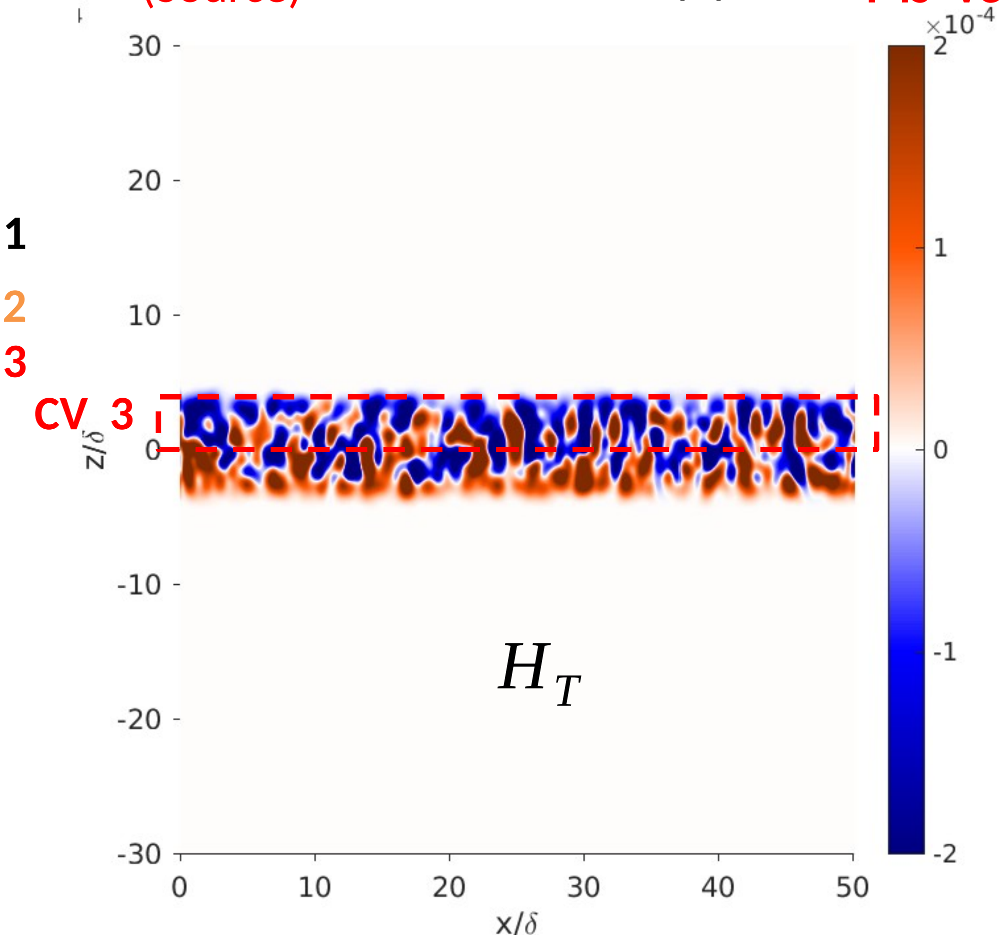


(Front) $\quad Control \;\; Vol \; 1: \;\; \frac{\partial h}{\partial t} \approx -\nabla \cdot [\mathbf{F}]$ **Div. -ve** **F is -ve**

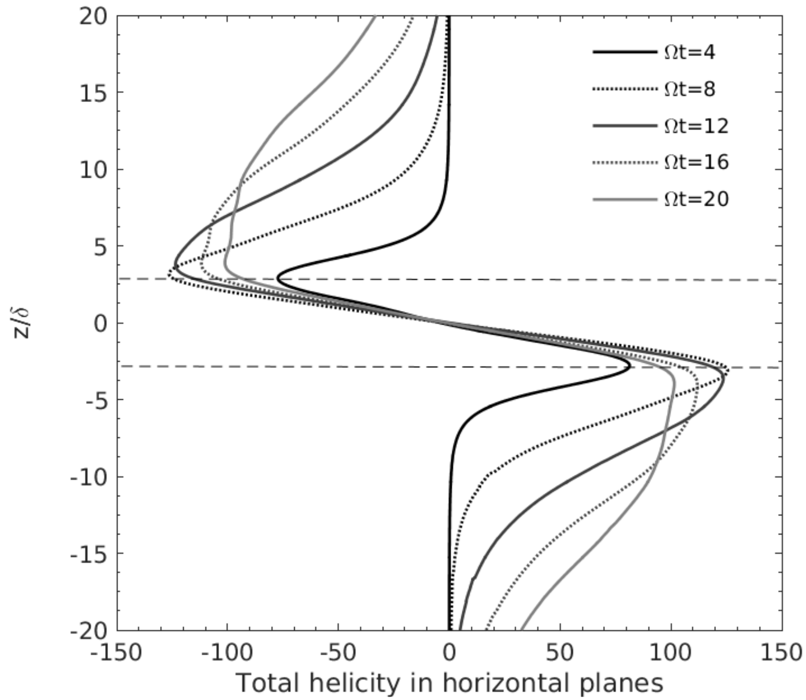
(Flux) $\quad Control \;\; Vol \; 2: \;\; \nabla \cdot [\mathbf{F}] \approx 0$

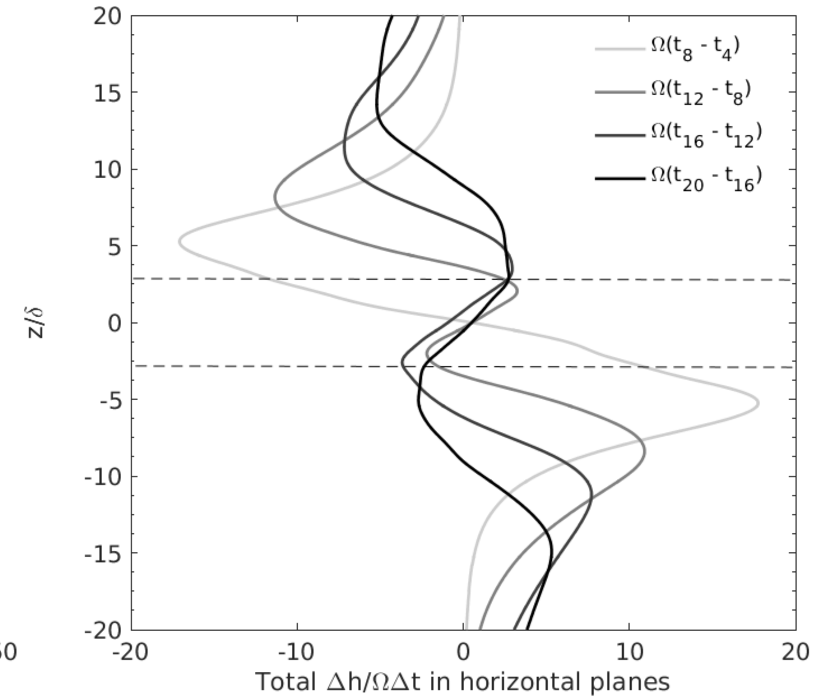
(Source) $\quad Control \;\; Vol \; 3: \;\; \nabla \cdot [\mathbf{F}] \approx H_T$ **Div. +ve** **F is -ve**

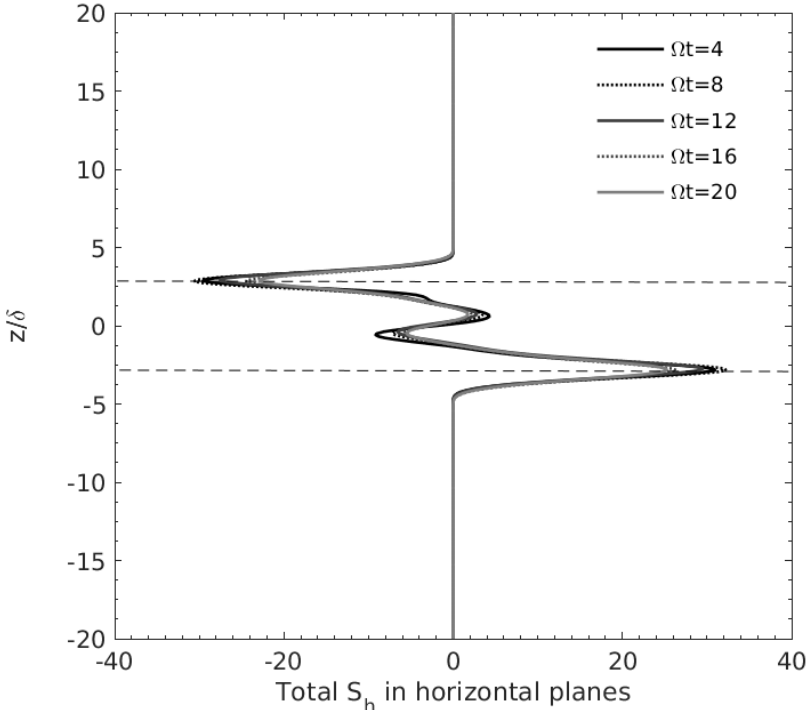




$$\frac{\partial h}{\partial t} = -\nabla \cdot [\mathbf{F}] + H_T$$







(Front) $\textit{Control Vol 1}: \quad \frac{\partial h}{\partial t} \approx -\nabla \cdot [\mathbf{F}]$

(Flux) $\textit{Control Vol 2}: \quad \nabla \cdot [\mathbf{F}] \approx 0$

(Source) $\textit{Control Vol 3}: \quad \nabla \cdot [\mathbf{F}] \approx H_T$

Summary & Speculations

The thermal 'source' of helicity **spontaneously** acquires the **same** sign as that of the helicity due to an **interplay** between the wave/velocity field and the buoyancy field.

Perhaps the same phenomena happens in the geodynamo/ non-magnetic simulations but in a statistical sense!

More work needed to understand the flux of helicity in spherical simulations.

Similar model problems can be designed (e.g. with magnetic field) to probe further

Thank You

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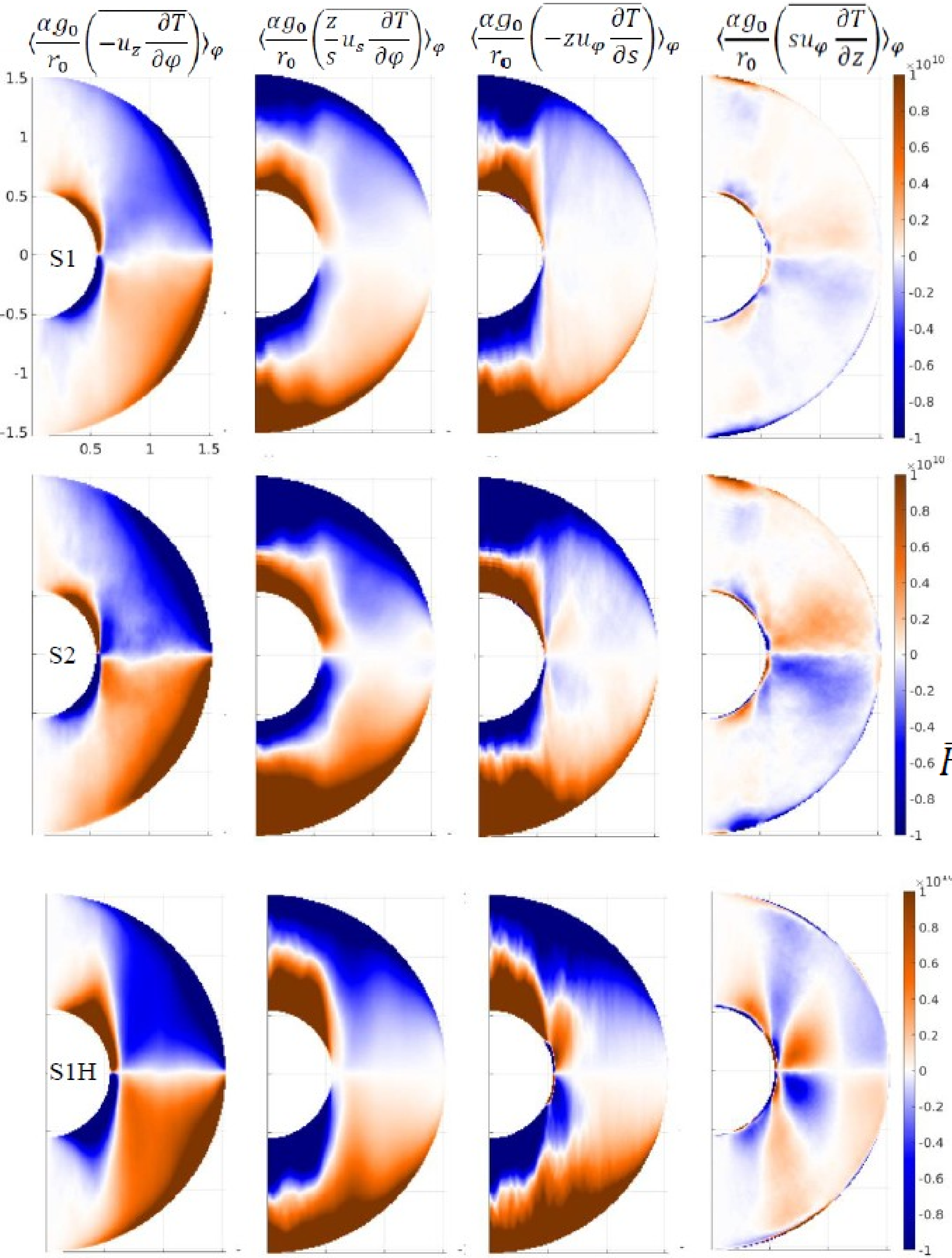
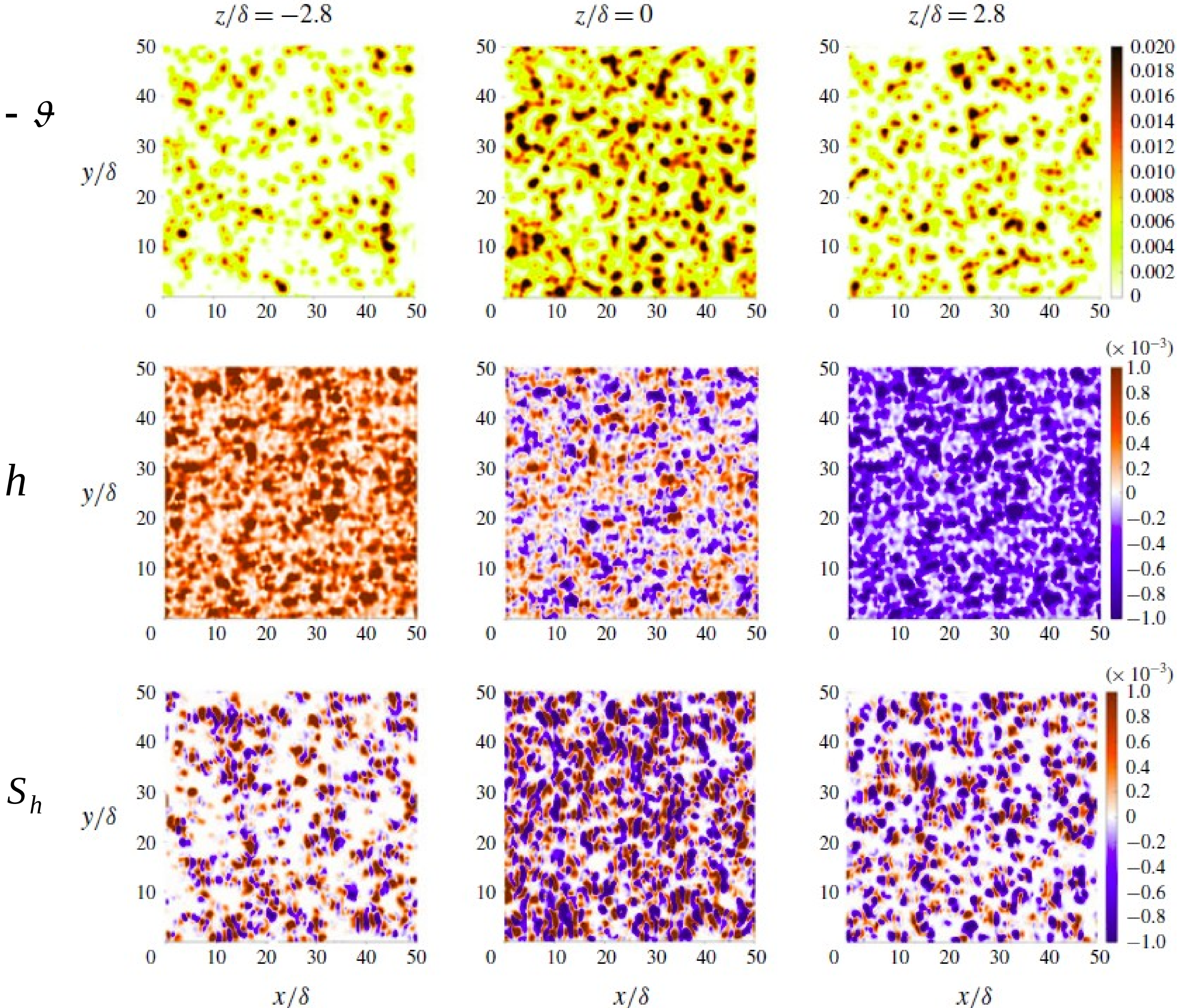


$$H_T = H_{T1} + H_{T2}$$

$$H_{T1} = \nabla \cdot \left(\boldsymbol{u} \times (-\alpha T \mathbf{g})\right)$$

$$H_{T2} = -2\alpha\, \boldsymbol{u} \cdot (\nabla T \times \mathbf{g})$$

$$\overline{H}_{T2} = \frac{2\alpha g_0}{r_0}\left(\overline{(zu_s - su_z)\frac{\partial T}{s\partial\varphi} + u_\varphi\left(s\frac{\partial T}{\partial z} - z\frac{\partial T}{\partial s}\right)}\right)$$



$z/\delta = -2.8$
$z/\delta = 0$
$z/\delta = 2.8$
$-\vartheta$
h
S_h
y/δ
x/δ
$(\times 10^{-3})$

MagIC: numerical details

Decomposing the velocity and magnetic field vectors into poloidal (W) and toroidal (Z) potentials,

$$\boldsymbol{u}(r,\theta,\phi) = \nabla\times\nabla\times[\hat{\boldsymbol{e}}_r W(r,\theta,\phi)] + \nabla\times[\hat{\boldsymbol{e}}_r Z(r,\theta,\phi)]$$
$$\boldsymbol{B}(r,\theta,\phi) = \nabla\times\nabla\times[\hat{\boldsymbol{e}}_r g(r,\theta,\phi)] + \nabla\times[\hat{\boldsymbol{e}}_r h(r,\theta,\phi)]$$

$$\hat{\boldsymbol{e}}_r\cdot\boldsymbol{u} = -\Delta_H W$$
$$\hat{\boldsymbol{e}}_r\cdot(\nabla\times\boldsymbol{u}) = -\Delta_H Z,$$

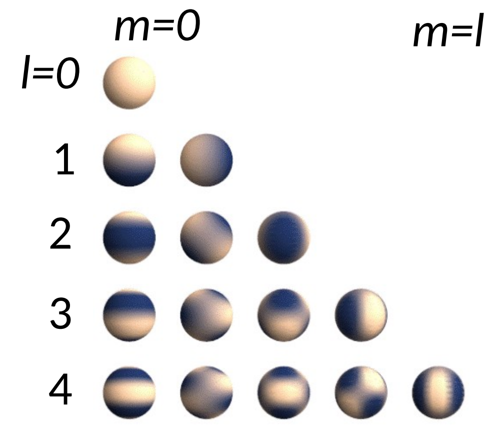
where, Δ_H is the horizontal part of the Laplacian,

$$\Delta_H = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial^2\phi}.$$

In terms of W and Z, the velocity can be written as

$$\boldsymbol{u}(r,\theta,\phi) = -(\Delta_H W)\hat{e}_r + \left(\frac{1}{r}\frac{\partial W}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial Z}{\partial\phi}\right)\hat{e}_\theta + \left(\frac{1}{r\sin\theta}\frac{\partial W}{\partial\phi} - \frac{1}{r}\frac{\partial Z}{\partial\theta}\right)\hat{e}_\phi.$$

The scalars can be expanded in spherical harmonics,



$$W(r,\theta,\phi) = \sum_{l,m} W_l^m(r) Y_l^m(\theta,\phi)$$
$$Z(r,\theta,\phi) = \sum_{l,m} Z_l^m(r) Y_l^m(\theta,\phi)$$

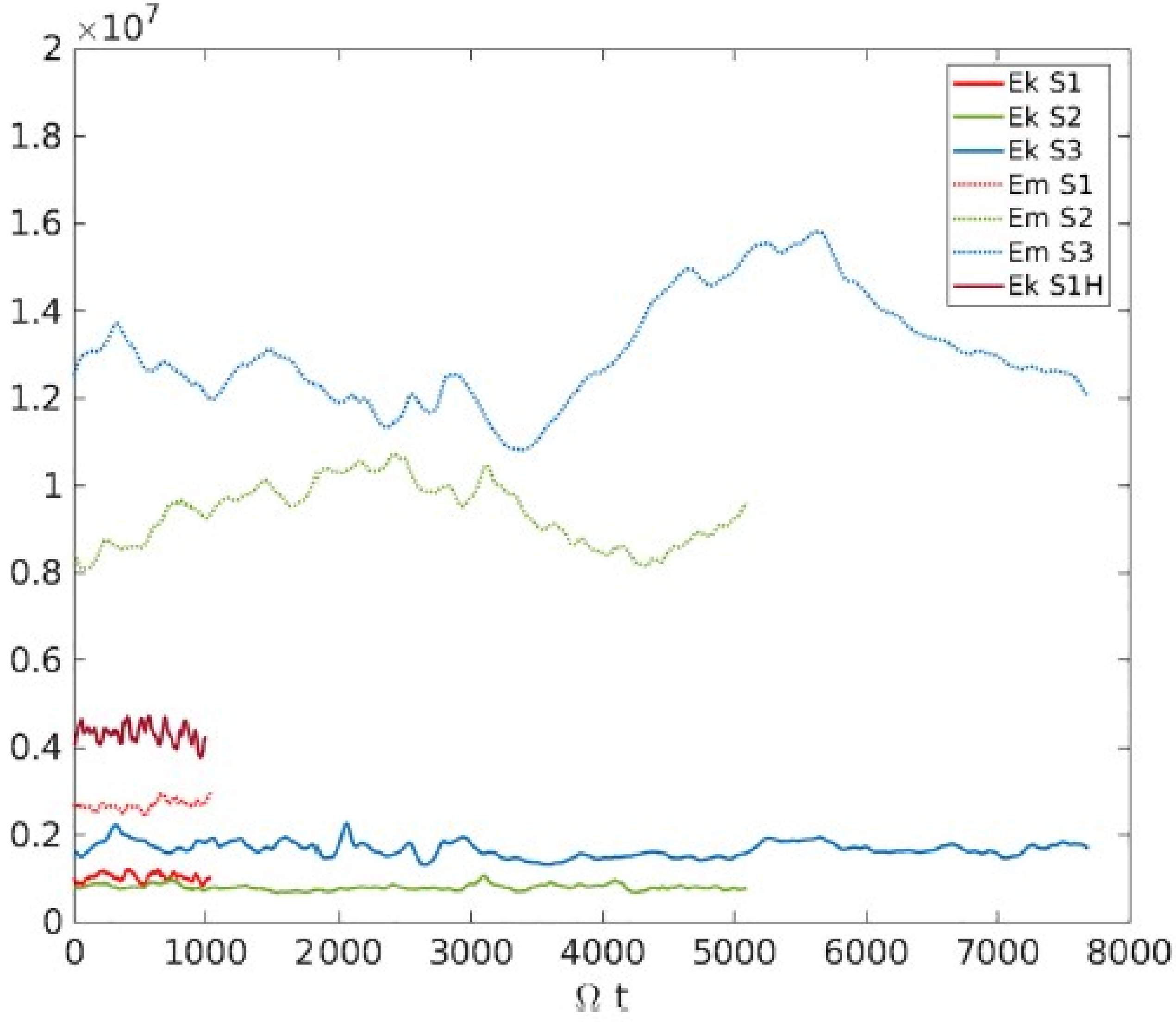
, where l and m denote the spherical harmonic degree and order, respectively, and $Y_l^m(\theta,\phi) = P_l^m(\cos\theta)e^{im\phi}$, $P_l^m(\cos\theta)$ is Legendre Polynomial and $\sum_{l,m} = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l}$.

Conversely,

$$W_l^m(r) = \frac{1}{\pi}\int_0^{\pi} W_m(r,\theta) P_l^m(\cos\theta)\sin\theta d\theta$$
$$W_m(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} W(r,\theta,\phi)e^{-im\phi}d\phi$$

The potentials can be expanded in radial direction terms of Chebyshev polynomials ($C_n(r) = \cos[n \arccos(r)]$), where n is the radial index, as

$$W_l^m(r) = \sum_{n=0}^{N} W_{lmn} C_n(r).$$



×10^7
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1000
2000
3000
4000
5000
6000
7000
8000
Ω t
Ek S1
Ek S2
Ek S3
Em S1
Em S2
Em S3
Ek S1H