Effects of P-noninvariance in the magnetic field generation of the celestial bodies

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IZMIRAN

Magnetic Reynolds number

$$R_m = \frac{vl}{\nu_m}$$

$$\nu_m = \frac{c^2}{\sigma}$$

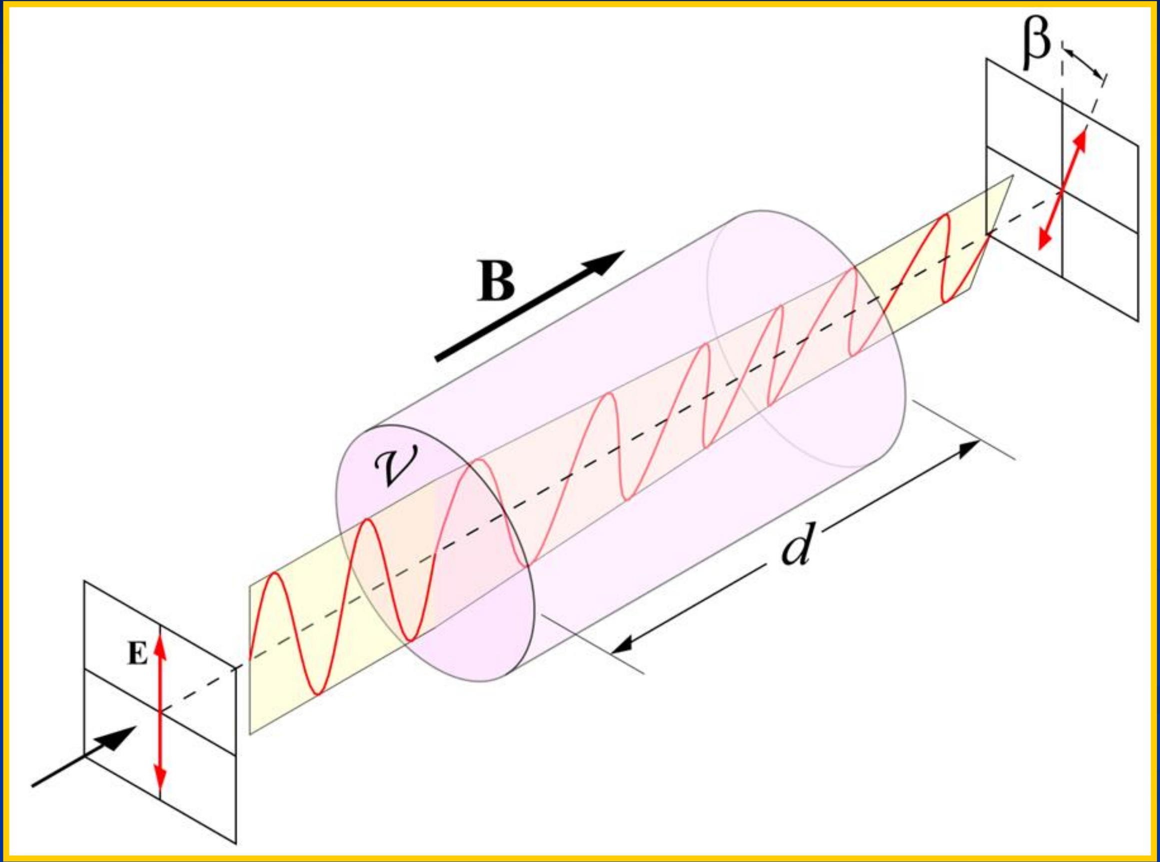
$$R_m \sim 10 - 100$$

- *laboratory*

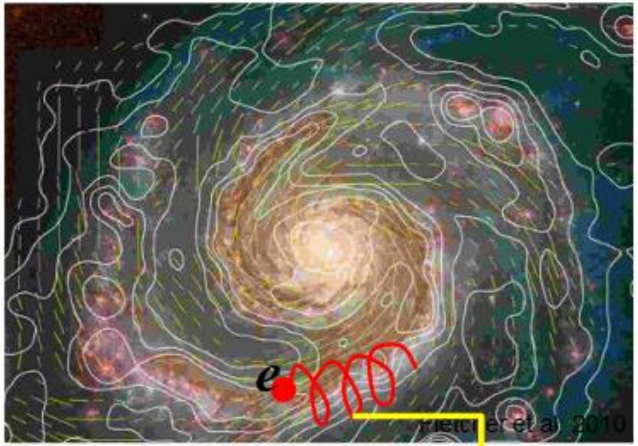
$$R_m = 10^6 - 10^8$$

- *stars*

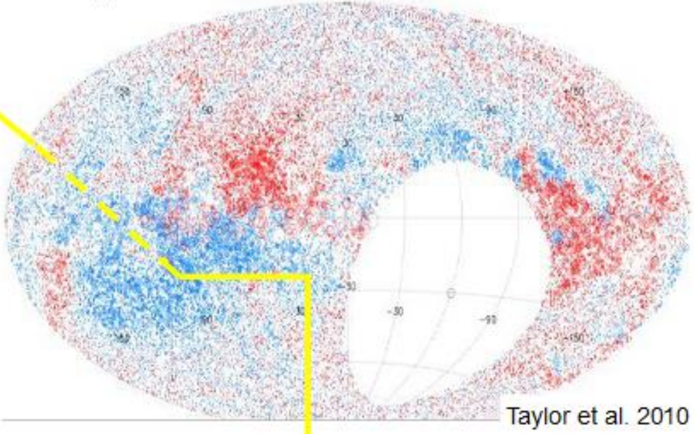
Rotation of the polarization plane. The Faraday effect



How to measure magnetic field?



γ



Taylor et al. 2010

A. Put charged particles in it and make them radiate synchrotron radiation....

$$E_\gamma \sim BE_e^2$$

B. Take polarized (radio) beam and make it propagate through the magnetic field....

$$RM \sim \int_{l.o.s} n_e(\vec{B} \cdot d\vec{l})$$

$$\beta = RM \cdot \lambda^2$$

Density of relativistic electrons too low in the IGM; Synchrotron emission too weak.

Density of free electrons in the IGM is too low.

Bounds on the cosmological magnetic fields (t_{now})

Upper bound:

1) $B < 10^{-9} - 10^{-10}\ G$ (Ruzmaikin & Sokoloff, 1977) from Faraday RM (subtracting the Milky Way contribution), $L >> L_{gal}$;

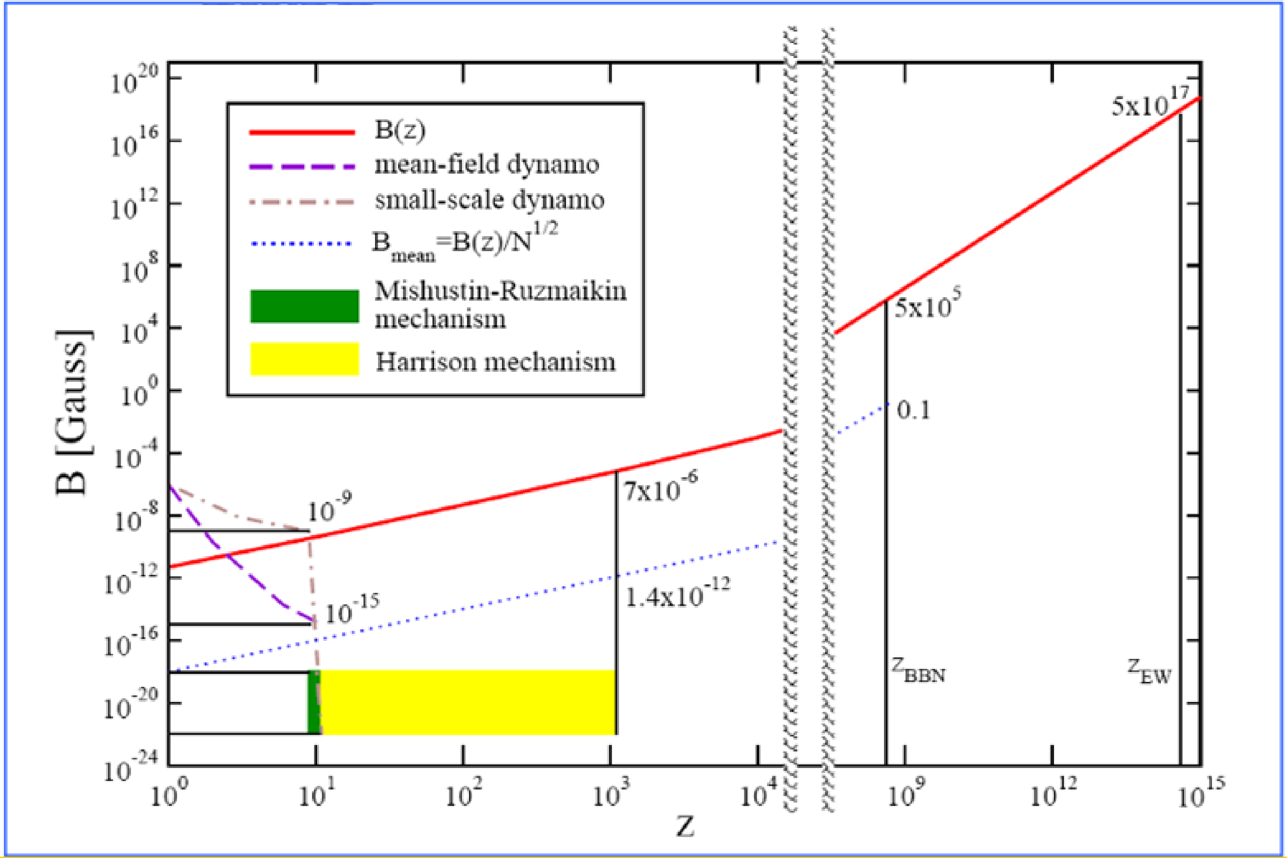
2) $B < 10^{-8} - 10^{-9}\ G$ (Barrow, Ferreira, Silk, 1997) from CMB anisotropy + uniform fiels at the start t_{rec}.

Lower bound:

$B > 10^{-16}\ G$ if $\lambda_B << D_e$

$B > 10^{-18}\ G$ if $\lambda_B >> D_e$ (Neronov, D. Semikoz, 2009), (Neronov, Vovk, 2010).

Magnetic field evolution after EWPT



The helicity parameter α in the excitation term $\nabla \times (\alpha\vec{\mathbf{B}})$ of the Faraday equation

$$\frac{\partial\vec{\mathbf{B}}}{\partial t} = \nabla \times \vec{v} \times \vec{\mathbf{B}} + \frac{1}{\sigma}\nabla^2\vec{\mathbf{B}} + \nabla \times (\alpha\vec{\mathbf{B}}) \quad (*)$$

The standard MHD. P-invariance violation.

$$\alpha = \frac{\tau_{corr}}{3} < \vec{u}(\nabla \times \vec{u}) >$$ – *pseudoscalar*

$$P\alpha P^{-1} \mapsto -\alpha$$ – *P - odd*

$$\vec{v} = \vec{\Omega} \times \vec{r} + \vec{u}$$

$$< \vec{u} >= 0$$

$$\vec{j} = \alpha \vec{B}, \vec{j} || \vec{B}$$

$$P\vec{\mathbf{B}}P^{-1} \mapsto \vec{\mathbf{B}}$$

Equation (*) is P- even,

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \nabla \times \vec{v} \times \vec{\mathbf{B}} + \frac{1}{\sigma}\nabla^2 \vec{\mathbf{B}} + \nabla \times (\alpha \vec{\mathbf{B}})$$

Anomalous MHD

$\hbar = c = 1$

$$\nabla \times \vec{\mathbf{B}} = \overrightarrow{J_{Ohm}} + \overrightarrow{J_{anom}}$$

$$\overrightarrow{J_{Ohm}} = \sigma(\vec{\mathbf{E}} + \vec{v} \times \vec{\mathbf{B}}) \qquad \overrightarrow{J_{anom}} = \frac{e^2}{2\pi^2}\mu_5 \vec{\mathbf{B}}$$

Vector currents: $P\vec{J}P^{-1} \mapsto -\vec{J}$

$\alpha = \frac{e^2 \mu_5}{2\sigma\pi^2}$ - *pseudoscalar,* $P\alpha P^{-1} \mapsto -\alpha \quad \mu_5 = \frac{\mu_R - \mu_L}{2}$

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \nabla \times \vec{v} \times \vec{\mathbf{B}} + \frac{1}{\sigma}\nabla^2 \vec{\mathbf{B}} + \nabla \times (\alpha \vec{\mathbf{B}})$$

Parity violation in electroweak interactions.
Weinberg-Salam Standard Model

$$\alpha = \frac{\ln 2}{4\sqrt{2}\,\pi^2}\left(\frac{G_F T}{\lambda^{\nu}_{fluid}}\right)\left(\frac{n_\nu - n_{\overline{\nu}}}{n_\nu}\right)$$ - *scalar*

$$n_{\nu,\bar{\nu}} = \frac{n_e}{2} \qquad n_e = 0.183T^3, T >> m_e$$

CP symmetry is conserved in weak interactions

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \frac{1}{\sigma}\nabla^2 \vec{\mathbf{B}} + \nabla \times (\alpha \vec{\mathbf{B}})$$

CP-parity conservation in the MHD-equations accounting for weak interactions

$$(\mathrm{CP})\,(n_{\nu\mathrm{L}} - n_{\bar{\nu}\mathrm{R}})\,(\mathrm{CP})^{-1} \longrightarrow -(n_{\nu\mathrm{L}} - n_{\bar{\nu}\mathrm{R}}),$$

since $n_{\nu\mathrm{L}} \leftrightarrow n_{\bar{\nu}\mathrm{R}}$, or $\alpha \longrightarrow -\alpha$.

As a result, CP – parity is conserved in the Faraday equation, for the

magnetic field itself is CP - odd: $(\mathrm{CP})\,\mathbf{B}\,(\mathrm{CP})^{-1} \rightarrow -\mathbf{B}$.

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \frac{1}{\sigma}\nabla^2 \vec{\mathbf{B}} + \nabla \times (\alpha \vec{\mathbf{B}})$$

α^2 - dynamo in Early Universe

$$B(t) = B_0 exp\left(\int_{t_0}^{t} \frac{\alpha^2(\tau)}{4\eta(\tau)}\, d\tau\right)$$

$$T_0 = 20GeV << T_{EWPT} \simeq 100GeV$$

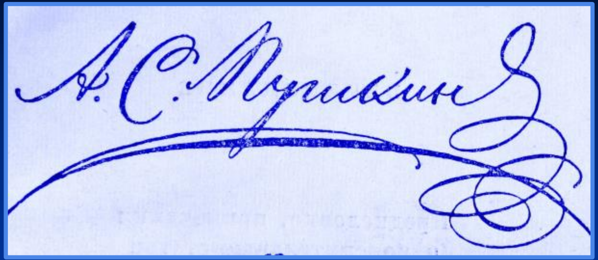
$$B(x) = B_0 exp\left[25\int_x^1 \left(\frac{\xi_{\nu_e}(x')}{0.07}\right)^2 (x')^{10} dx'\right]$$

$$t[sec] \simeq (T[MeV])^{-2},$$

$$for \quad x = T/T_0 \ll 1$$

$$|\xi_{\nu_e}| = \mu_{\nu_e}/T \leq 0.07$$ – *Dolgov et al., 2002*

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \frac{1}{\sigma}\nabla^2\vec{\mathbf{B}} + \nabla \times (\alpha\vec{\mathbf{B}})$$



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1777–1855 Carl Friedr. Gauß
YA9676920G8
Deutsche Bundesbank
Frankfurt am Main
1.Oktober 1993

Magnetic helicity

The definition: $H(t) = \int d^3x\, \mathbf{A}(\mathbf{x},t)\cdot\mathbf{B}(\mathbf{x},t)$ is the magnetic helicity

(Gauss was first who calculated the knots number m, $H = 2m\Phi_1\Phi_2$).

The topology number m shows the ***linkage and tangling*** of magnetic force lines and this is a good integral of motion in MHD: it is conserved much better (decays much slower) than the magnetic energy in viscous matter. It is also ***GAUGE-INVARIANT*** under transformation $\mathbf{A}(\mathbf{x},t) \to \mathbf{A}(\mathbf{x},t) + \nabla\chi$ and supports the evolution of magnetic field (via inverse cascade) to large-scale fields from the small-scale ones.

The change of helicity (in gauge $\mathbf{E} = -\partial\mathbf{A}/\partial t$) using also $\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$ is given by

$$\frac{dH}{dt} = -2\int d^3x\, \mathbf{E}\cdot\mathbf{B},$$

and in ideal plasma ($\sigma_{cond} = \infty$, standard MHD with Lorentz force only, $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$) helicity is conserved.

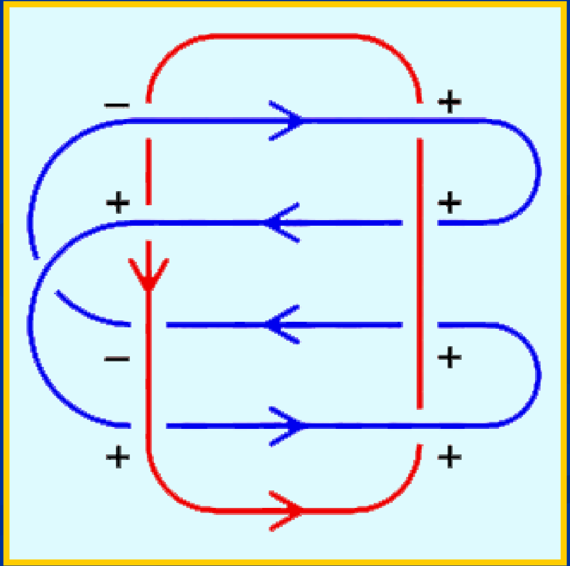
The magnetic helicity is a topological invariant

$$\frac{dH}{dt} = -2\int(\vec{E}\cdot\vec{B})dx^3 = -\frac{2}{\sigma}\int(\vec{j}\vec{B})dx^3$$

$$H = const$$

$$H = m\Phi_1\Phi_2$$

$$m = \pm 1, \pm 2, \ldots$$

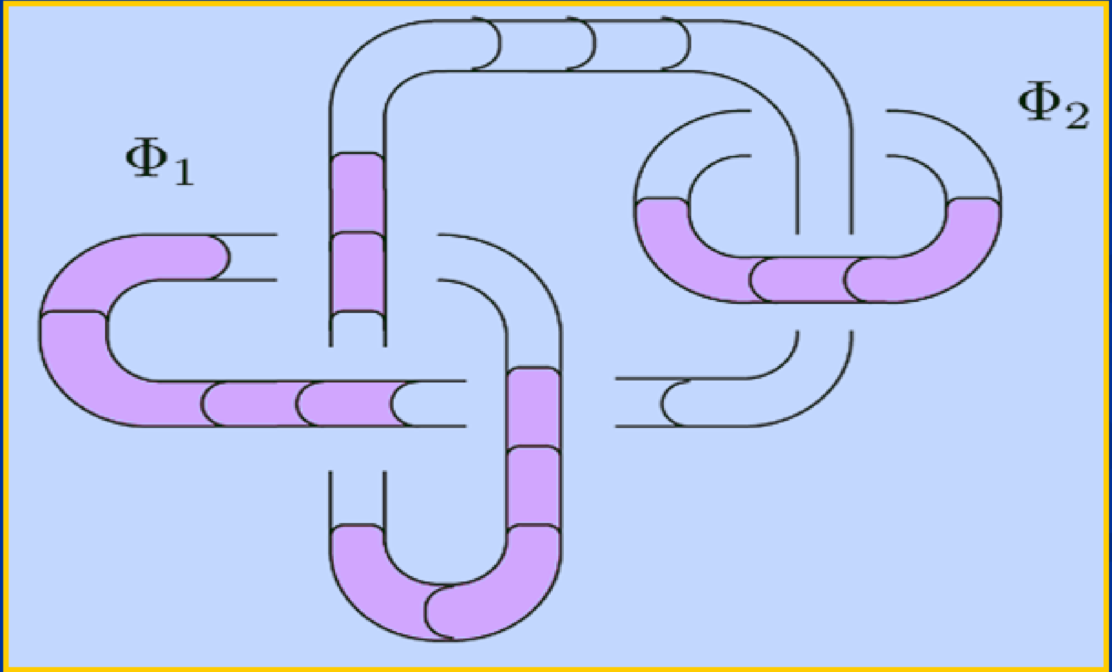


Φ_2

Φ_1

Hypermagnetic fluxes $\Phi = \int \mathbf{B_Y} \cdot d\mathbf{S}$ and topology (linkage) number m (Chern- Simons analogue)

$$H = \int_v d^3x(\mathbf{B}_Y \cdot \mathbf{Y}) = m\Phi_1\Phi_2$$



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Thank you for your attention!

Abelian anomaly in QED and conservation of the total helicity

Pseudovector current $J_5^\mu = \bar{\psi}_R \gamma^\mu \psi_R - \bar{\psi}_L \gamma^\mu \psi_L = \bar{\psi} \gamma^\mu \gamma^5 \psi$ is not conserved in electromagnetic fields. For massless fermions

$$\partial_\mu J_5^\mu = \frac{2\alpha_{em}}{\pi} \mathbf{E} \cdot \mathbf{B} \neq 0.$$

Integrating over volume, $V^{-1} \int d^3x(...)$ one gets

$$\frac{\mathrm{d}}{\mathrm{dt}}(n_R - n_L) = V^{-1} \left(\frac{2\alpha_{em}}{\pi}\right) \int d^3x \mathbf{E} \cdot \mathbf{B} = -\frac{\alpha_{em}}{\pi} \frac{\mathrm{d}h}{\mathrm{dt}},$$

where $h = V^{-1} \int d^3x \mathbf{A} \cdot \mathbf{B}$ is the magnetic helicity density.

In hot (ultrarelativistic) plasma $n_{R,L}(t) = \mu_{R,L}(t) T^2/6$, $\mu_R - \mu_L = 2\mu_5(t)$,

the **TOTAL** (particle+magnetic field) **HELICITY DENSITY is CONSERVED**,

$$\frac{\mathrm{d}}{\mathrm{dt}} \left(n_R - n_L + \frac{\alpha_{em}}{\pi} h\right) = 0.$$

Processes in the intergalactic medium (IGM)

1) $\gamma + \gamma_{EBL} \longrightarrow e^+ + e^-$

with a threshold of $\omega_{EBL} = m_e^2/E_\gamma$ = 0.25 eV for E_γ= 1 TeV,

or $\lambda_{EBL} > 5$ μm $> \lambda_{Red} = 0.7$ μm = 7000 Å;

2) The cascade, i.e. inverse Compton scattering (IC) on the relic photons taking into account the CMF for charged particles,

$$e^{\pm}(E) + \gamma_{CMB} \rightarrow e^{\pm}(E') + \gamma',$$

where $\omega' \sim$ 10 GeV – 100 GeV >> $\omega_{CMB} = 3 \cdot 10^{-4}$ eV

for $E = E_\gamma/2$ = 0.5 TeV.

Photon and charged particles mean free paths in the intergalactic medium

$$D_\gamma = 1/(\sigma_{\gamma\gamma}\, n_{EBL}) \;\; >> \;\; D_e = 3m_e^2/(4\sigma_T U_{CMB} E_e),$$

where the mean free path of the original photon relative to the reaction $\gamma + \gamma_{EBL} \longrightarrow e^+ + e^-$ is

$$D_\gamma = 80\varkappa(10\text{ TeV})/E_\gamma \sim \text{Mpc},$$

and the free path of the charged particles $e^\pm$ in the inverse Compton scattering (IC), $e^\pm + \gamma_{EBL} \longrightarrow e^\pm + \gamma'$ is

$$D_e = 10^{23}(10\text{ TeV})/E_e\ \text{cm} \sim 60\text{ kpc}.$$

for the electron (positron) energy $E_e = E_\gamma / 2 = 5$ TeV.

IGMF filling factor